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Source: *The Journal of Industrial Economics*, Vol. 40, No. 2 (Jun., 1992), pp. 229-231
Published by: Wiley
Stable URL: http://www.jstor.org/stable/2950513

THE JOURNAL OF INDUSTRIAL ECONOMICS 0022-1821 $2.00
Volume XL June 1992 No. 2

A COMMENT ON A MODEL OF VERTICAL PRODUCT DIFFERENTIATION

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In a duopoly model with vertical differentiation, if the firms do not cover the market, the lower quality firm chooses a quality level exactly 4/7 of that of the higher quality firm, and chooses a price which is 2/7 of the price of the higher quality firm.

In their pathbreaking paper, Shaked and Sutton [1982] demonstrate how the existence of quality differences relaxes price competition between competing firms, so that they command positive profits in equilibrium. Quality differences are formalized in terms of a framework for preferences due to Gabszewicz and Thisse [1979] in which individuals with identical preferences may, nevertheless, choose different goods because their respective marginal utilities of income differ. Tirole [1988, section 2.1] introduces an alternative utility function which captures the spirit of the earlier papers and yet leads to simpler solutions.

The purpose of this short note is to extend Tirole's discussion and to point to an explicit solution in the case where the firms do not cover the market. In this case, the lower quality firm chooses a quality which is exactly 4/7 of that of the higher quality firm, while its price is 2/7 of the price of the higher quality firm. Our paper does not offer any conceptual innovations, and is little more than a footnote to Tirole's discussion. However, the result may have some pedagogical value.

There are two firms, 1 and 2, which produce distinct goods, sold at prices p_1 and p_2 respectively. Each firm's product is associated with a number $v_i > 0$, $i \in \{1, 2\}$ which represents its quality level. There are no production costs.

There is a continuum of consumers distributed uniformly over the interval $[\theta - 1, \theta]$ with unit density, where $\theta > 1$. Each consumer does one of three things—buy from firm 1, buy from firm 2, or not buy at all. The consumer indexed by the parameter $\tilde{\theta} \in [\theta - 1, \theta]$ maximizes the following utility function.

$$u_{\tilde{\theta}} = \begin{cases} \tilde{\theta} v_i - p_i & \text{if he buys from firm } i, \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

This is the utility function described in Tirole [1988 p. 96]. All consumers prefer higher quality at a given price, but a consumer with higher $\tilde{\theta}$ is willing to pay more for higher quality.

Firms 1 and 2 play a three stage game. In the first stage, firm 1 chooses v_1

from the interval $[0, \bar{v}]$, where $\bar{v} > 0$. In the second stage, firm 2 chooses v_2 from the interval $[0, v_1]$, having observed v_1. Quality choice is costless. In the third stage, firms choose prices simultaneously, having observed the choices of v_1 and v_2.

A smaller θ has the interpretation of a greater diversity of tastes, and solutions fall under two cases. If θ is large, the firms cover the market, while if θ is small, tastes are sufficiently diverse so that some consumers do not buy from either firm. We shall be interested in this latter case.

Proposition. In the equilibrium in which the two firms do not cover the market, $v_2 = (4/7)v_1$ and $p_2 = (2/7)p_1$. The two firms do not cover the market if $\theta < 8/7$.

In solving the game, consider the demand faced by each firm. By the rules of the game, we have $0 \leqslant v_2 \leqslant v_1$. A consumer with index θ_2 for which $\theta_2 v_2 - p_2 = 0$ will be indifferent between buying from firm 2 and not buying at all. Any consumer with index greater than θ_2 will prefer to buy from firm 2 than not to buy at all. Analogously, a consumer with index θ_1, where $\theta_1 v_1 - p_1 = \theta_1 v_2 - p_2$ will be indifferent between buying from firm 1 and buying from firm 2. Any consumer with index greater than θ_1 will prefer to buy from firm 1 than from firm 2. Thus, if the two firms do not cover the market, the demand for each firm's product is given as follows. Let $p = (p_1, p_2)$, $v = (v_1, v_2)$.

$$\begin{aligned} D_1(p, v) &= \theta - \frac{p_1 - p_2}{v_1 - v_2} \\ D_2(p, v) &= \frac{p_1 - p_2}{v_1 - v_2} - \frac{p_2}{v_2} \end{aligned} \tag{2}$$

Since costs are zero, the profit function for firm i, $\pi_i(p, v)$, is given by $p_i D_i(p, v)$. Taking v_1 and v_2 as given, the best reply functions are obtained from the first order conditions, and are as follows.

$$\begin{aligned} p_1 &= \tfrac{1}{2}(p_2 + \theta(v_1 - v_2)) \\ p_2 &= \left[\frac{v_2}{2v_1}\right] p_1 \end{aligned} \tag{3}$$

Solving for p_1 and p_2 in terms of v_1, v_2 and θ,

$$\begin{aligned} p_1 &= \frac{2\theta v_1 (v_1 - v_2)}{4v_1 - v_2} \\ p_2 &= \frac{\theta v_2 (v_1 - v_2)}{4v_1 - v_2} \end{aligned} \tag{4}$$

Anticipating the price competition in the third stage, the demand functions can be expressed in terms of v_1 and v_2. Substituting (4) into (2),

$$
\begin{aligned}
D_1(v) &= \frac{2\theta v_1}{4v_1 - v_2} \\
D_2(v) &= \frac{\theta v_1}{4v_1 - v_2}
\end{aligned} \tag{5}
$$

The profit functions can then be expressed as follows.

$$
\begin{aligned}
\pi_1(v) &= \frac{4\theta^2 v_1^2 (v_1 - v_2)}{(4v_1 - v_2)^2} \\
\pi_2(v) &= \frac{\theta^2 v_1 v_2 (v_1 - v_2)}{(4v_1 - v_2)^2}
\end{aligned} \tag{6}
$$

The first order condition for the maximization of π_2 yields;

$$(4v_1 - v_2)(v_1 - 2v_2) + 2v_2(v_1 - v_2) = 0 \tag{7}$$

which simplifies to $v_2 = (4/7)v_1$. π_1 is then proportional to v_1 so that firm 1 sets $v_1 = \bar{v}$ in stage 1. Thus, in equilibrium, we have $v_1 = \bar{v}$, $v_2 = (4/7)\bar{v}$, $p_1 = \theta\bar{v}/4$, and $p_2 = \theta\bar{v}/14$.

In order to complete the solution, it remains to check that the two firms do not cover the market if $\theta < 8/7$. The consumer with the lowest index does not buy from either firm if $\theta - 1 < p_2/v_2$. But $p_2/v_2 = \theta/8$. Thus, the firms do not cover the market if $\theta - 1 < \theta/8$, or $\theta < 8/7$. This completes the solution.

It is worth noting that the solution is independent of the parameter θ provided that $\theta < 8/7$. The maxim for an entrant (at least in this example) seems to be: "choose a quality level which is just over half that of the established firm". In the ensuing price game, the ratio of prices of high to low quality products is twice that of the ratio of qualities.

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ACCEPTED JUNE 1991

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