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# Government spending and the exchange rate\*

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## Abstract

Contrary to widespread empirical evidence, standard NOEM models imply that the real exchange rate appreciates following an increase in public spending. This paper introduces productive government purchases and shows that the real exchange rate can depreciate after a positive spending shock, thus reconciling the theoretical model with the empirical evidence. Under empirically consistent parameterization, the model implies a depreciation both on impact and in the transition. The transmission mechanism works through an increase in domestic private-sector productivity, spurred by government purchases, which reduce domestic real marginal costs.

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# 1 Introduction

The financial crisis that led the world economy into a global recession in 2009 stimulated a revived interest in the role of fiscal policy as a stabilization tool. Relevant discretionary fiscal interventions have been undertaken in the US and in many other industrialized countries, often coupled with expansionary monetary policies. Interestingly, however, although the effects of fiscal shocks and their international transmission have long been investigated in the literature, not much consensus has been achieved. One reason is that Ricardian equivalence holds in the benchmark Redux model (Obstfeld and Rogoff, 1995) that started the so called New Open Economy Macroeconomics (NOEM), which limits the range of fiscal policies that can be analyzed.

Recently, the standard two-country framework based on the Representative Agent (RA) model has been extended to account for agents' heterogeneity, turnover in financial markets and some form of market imperfection or incompleteness that allow to depart from Ricardian equivalence and investigate fiscal policies more in detail (see Cavallo and Ghironi, 2002, Ganelli, 2005b, Galí, López-Salido and Vallés, 2007, Leith and Von Thadden, 2008, Di Giorgio and Nisticò, 2007, 2013). Thanks to these advances it is possible to make a comparison between the outcomes of fiscal shocks in traditional models such as the static Mundell Fleming version of IS-LM and the intertemporal approach used in the modern literature. These new contributions allow for a better understanding of the role played by different factors in affecting the response of the exchange rate, a key variable for shaping the international transmission and propagation of shocks.

In this paper, we develop a dynamic stochastic general equilibrium (DSGE) two-country model which is able to highlight the key mechanisms affecting the exchange-rate response of different fiscal shocks. The model extends to a fully specified dynamic and stochastic setting the intuition of Basu and Kollmann (2013), where government spending is allowed to affect private-sector productivity: a fraction of government spending is used to accumulate productive public capital that spills over to labor productivity in the private sector. Moreover, by adopting on the demand side a “perpetual youth” structure, along the lines of Di Giorgio and Nisticò (2007, 2013), we break Ricardian equivalence so as to analyze both balanced-budget and debt-financed fiscal policy.

Our main result is that productive public capital is key to determine the short-run and the long-run effects of government spending shocks on the real exchange rate. Specifically, we show that an exogenous increase in public spending has a double effect on real marginal costs. On the one hand, with higher demand and sticky prices, real wages must increase, implying higher real marginal costs. On the other hand, by improving labor productivity,

firms become more competitive and real marginal costs decline. Depending on which effect is dominant, we have higher or lower inflation, triggering, through the interest rate set by the central bank an appreciation or a depreciation. We show that the second effect is stronger for a reasonable calibration, thereby depreciating the real exchange rate, both on impact and in the medium run, after a balanced-budget increase in public spending. When the increase in public spending is unbalanced, the on-impact response of the exchange rate depends on the general fiscal regime: when the government follows a countercyclical primary-deficit rule that responds to public debt, the exchange rate depreciates, while in case of an exogenous tax rule it may appreciate. In both cases, however, the exchange rate depreciates in the medium run, consistently with the empirical evidence. On the contrary, an expansionary fiscal policy conducted by means of a tax cut appreciates the exchange rate, as in this case there is no positive productivity spillover on the private sector and marginal costs increase because of higher demand.

Our model overcomes a well known criticism of the basic DSGE open economy model in which, contrary to most of the empirical evidence, the exchange rate appreciates following a positive fiscal shock. By assuming that a plausible fraction of government spending affects private-sector productivity, we obtain a significant depreciation of the real exchange rate following a positive shock in government consumption. We also highlight which features in the standard macroeconomic model have an impact on the exchange rate response, particularly the degree of home bias in government consumption and the degree of endogeneity of monetary policy.<sup>1</sup>

Our paper is linked to the large literature on the effects of fiscal policy in open economies. From a theoretical perspective, the starting point is the *Redux* model *à la* Obstfeld and Rogoff (1995), in which the exchange rate depreciates after a balanced-budget fiscal expansion. In the same setting, Ganelli (2005a) shows that if government consumption is fully home biased, the exchange rate is unaffected by the fiscal expansion, given that the expansionary effect on domestic demand is completely offset by the required increase in taxes. More recently, Di Giorgio, Nisticò and Traficante (2015) show that, in the *Redux* framework, the nominal exchange rate can even appreciate in response to positive spending shocks, if public spending is home-biased and monetary policy is endogenous. Given that an increase in public spending is more expansionary at home than abroad, endogenous and countercyclical monetary policy triggers a relatively stronger monetary restriction at home, which appreciates the exchange rate. This result bridges the original *Redux* model with more modern DSGE open-economy models such as Corsetti and Pesenti (2001) and Devereux and Engel (2003). In these models,

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<sup>1</sup>This analysis is consistent to the one in Di Giorgio et al. (2015) that is able to reconcile, in a generalized version of the *Redux* model, the apparently different results of Obstfeld and Rogoff (1995), Mundell and Fleming (1956) and Ganelli (2005a).

indeed, a balanced-budget fiscal expansion leads to an appreciation of the exchange rate.

Recent empirical evidence shows, however, that the exchange rate indeed depreciates after a positive fiscal shock (see Kim and Roubini (2008), Monacelli and Perotti (2010), Beetsma et al. (2008) and Bénétrix and Lane (2013)), calling into question the assumptions underlying modern NOEM models. A few theoretical models have proposed mechanisms that induce an exchange rate depreciation in the face of a balanced-budget fiscal expansion. Kollmann (2010) introduces incomplete financial markets and finds that, after an increase in government spending, a negative wealth effect induces households to work more. As a consequence, domestic output increases, the terms of trade worsen and the real exchange rate depreciates. Corsetti, Meier and Müller (2012) show that the effects of fiscal shocks are deeply related to the expectations about future policy adjustment. With spending reversals, higher government consumption leads to an immediate decline in long-term real interest rates and a real exchange rate depreciation. Basu and Kollmann (2013) introduce productive government spending in a static frictionless model. They show that a real depreciation can arise from the interaction between the risk-sharing condition and the positive supply-side effect of productive government purchases.<sup>2</sup> We follow their insight and build it into a fully-fledged dynamic open-economy model with nominal rigidities and non-Ricardian agents. This framework allows us to evaluate the dynamic effects of different fiscal policies, not only of balanced-budget increases in public spending. The assumption of productive public capital implies that government spending has a positive externality on private-sector productivity. Several studies provide empirical support to this assumption, although a general consensus has yet to be reached about the exact dimension of the effect (see Aschauer, 1989, Kamps, 2006, and Bom and Ligthart, 2009). The calibration we adopt in the numerical illustration of the model's implications is consistent with the values obtained in the literature.

The remaining of the paper is organized as follows. In Section 2, we describe a fully specified non-Ricardian two-country DSGE model of the business cycle. In Section 3 we provide a numerical illustration of the effects of balanced and non-balanced-budget fiscal shocks on key macroeconomic variables. Section 4 concludes.

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<sup>2</sup>Iwata (2013) estimates a medium-scale open economy DSGE model for Japan, providing evidence of both productive public capital and Edgeworth complementarity between private and public consumption. Impulse-response analysis of the estimated model predicts mild real exchange rate depreciations in the medium run after government spending shocks, and ambiguous responses in the short run.

## 2 The Model

The world economy consists of two structurally symmetric countries,  $H$  and  $F$ , of equal size.<sup>3</sup> Households, in each country, supply labor inputs to firms and demand a bundle of consumption goods consisting of both home and foreign goods. The productive sector produces a continuum of perishable goods, in the interval  $[0, 1]$ , which are differentiated across countries and with respect to one another. As in the modern NOEM and Dynamic New-Keynesian tradition, there are nominal rigidities in the form of a Calvo (1983) price-setting mechanism and monetary policy is specified as the control of a short-term interest rate through a Taylor-type feedback rule.<sup>4</sup>

In order to be able to analyze a broader range of fiscal shocks and compare our results with existing literature, moreover, we extend the standard framework to also break Ricardian equivalence through a perpetual-youth structure of the demand side of the economy. The general model is therefore a two-country OLG economy, along the lines of Di Giorgio and Nisticò (2007, 2013).

### 2.1 The Demand Side

The demand-side of our economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965). Each period, in each country, a constant share  $\gamma$  of traders in the financial markets is randomly replaced by newcomers with zero-financial wealth; from that period onward, these newcomers start trading in the financial markets and face a constant probability  $\gamma$  of being replaced as the next period begins.

Consumers have log-utility preferences over consumption and leisure, supply labor services in a domestic competitive labor market and demand consumption goods. Consequently, each domestic household belonging to cohort  $j$  supplies labor inputs ( $L$ ) to firms and demands consumption goods  $C$  in order to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - \gamma)^t \left[ \log C_t(j) + \delta \log(1 - L_t(j)) \right]$$

The consumption index for a household belonging to cohort  $j$ , is a CES bundle of domestic

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<sup>3</sup>This assumption allows us to economize on notation, without any qualitative effect on the results.

<sup>4</sup>See, among others, Benigno (2004) and Galí and Monacelli (2005).

and imported goods:

$$C(j) = \left[ \kappa^{\frac{1}{\theta}} C_H(j)^{\frac{\theta-1}{\theta}} + (1 - \kappa)^{\frac{1}{\theta}} C_F(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (1)$$

$$C^*(j) = \left[ (1 - \kappa)^{\frac{1}{\theta}} C_H^*(j)^{\frac{\theta-1}{\theta}} + \kappa^{\frac{1}{\theta}} C_F^*(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

where  $\theta > 0$  measures the elasticity of substitution between Home and Foreign goods. To allow for endogenous fluctuations in the real exchange rate, we introduce home bias in consumption:  $\kappa > 0.5$ .

The consumption sub-indexes  $C_i(j)$  and  $C_i^*(j)$  result from Dixit-Stiglitz aggregation of the goods produced in the two countries, with elasticity of substitution  $\epsilon > 1$ :

$$C_i(j) = \left[ \int_0^1 C_i(k, j)^{\frac{\epsilon-1}{\epsilon}} dk \right]^{\frac{\epsilon}{\epsilon-1}} \quad C_i^*(j) = \left[ \int_0^1 C_i^*(k, j)^{\frac{\epsilon-1}{\epsilon}} dk \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (3)$$

for each  $i = H, F$  indexing the two countries and each  $k = h, f$  indexing the continuum of differentiated goods, with  $h, f \in [0, 1]$ .

Total expenditure minimization yields the consumption-based price indexes for the goods produced in the two countries

$$P_i = \left[ \int_0^1 P_i(k)^{1-\epsilon} dk \right]^{\frac{1}{1-\epsilon}} \quad P_i^* = \left[ \int_0^1 P_i^*(k)^{1-\epsilon} dk \right]^{\frac{1}{1-\epsilon}} \quad (4)$$

for each  $i = H, F$  and each  $k = h, f$ , and the respective consumer-price indexes (CPI)

$$P = \left[ \kappa P_H^{1-\theta} + (1 - \kappa) P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad P^* = \left[ (1 - \kappa) P_H^{*1-\theta} + \kappa P_F^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (5)$$

In the equations above,  $P_i(k)$  and  $P_i^*(k)$  are the prices of the generic brand  $k$  – produced by the country  $i$  – denominated in the currency of country Home and Foreign, respectively. We assume that prices of the differentiated goods are set in the producer's currency (Producer Currency Pricing, PCP), and that the Law of One Price (LOP) holds:

$$P_i(k) = \mathcal{E} P_i^*(k) \quad P_i = \mathcal{E} P_i^*,$$

for each  $k = h, f$  and  $i = H, F$ , where  $\mathcal{E}$  is the nominal exchange rate defined as the domestic price of foreign currency. Therefore, equations (4) imply that  $P_H = \mathcal{E} P_H^*$  and  $P_F = \mathcal{E} P_F^*$ . However, equations (5) show that, since Home and Foreign agents preferences are not necessarily identical, there can be deviations from purchasing power parity (PPP)

unless  $\kappa = 0.5$  that is,  $P \neq \mathcal{E}P^*$ . We measure the deviations from PPP through the real exchange rate, defined as  $Q \equiv \mathcal{E}P^*/P$ . Moreover, we define the Terms of Trade (ToT) as the relative price of foreign goods in terms of home goods ( $\mathcal{S} \equiv P_F/P_H = P_F^*/P_H^*$ ).

Accordingly, the brand-specific demand for good  $h$ , produced in country  $H$  is

$$\begin{aligned} Y_H(h) &\equiv C_H(h) + C_H^*(h) + G_H(h) = \\ &= \left( \frac{P_H(h)}{P_H} \right)^{-\epsilon} \left[ \kappa \left( \frac{P_H}{P} \right)^{-\theta} C + (1 - \kappa) \left( \frac{P_H^*}{P^*} \right)^{-\theta} C^* + G \right], \end{aligned} \quad (6)$$

while that for good  $f$  produced in country  $F$  is

$$\begin{aligned} Y_F^*(f) &\equiv C_F(f) + C_F^*(f) + G_F^*(f) = \\ &= \left( \frac{P_F^*(f)}{P_F^*} \right)^{-\epsilon} \left[ (1 - \kappa) \left( \frac{P_F}{P} \right)^{-\theta} C + \kappa \left( \frac{P_F^*}{P^*} \right)^{-\theta} C^* + G^* \right], \end{aligned} \quad (7)$$

where in each country government is assumed to consume an exogenously given amount of national goods (see below for details).

Households can allocate savings among a full set of domestic state-contingent private securities and two internationally traded riskless zero-coupon nominal bonds  $\tilde{B}_{i,t}$  issued in the two currencies by the governments to finance their budget deficits. In each country consumers are endowed with an equal amount of non-tradable shares of the domestic firms. Therefore, the budget constraint for each domestic household belonging to cohort  $j$  reads as

$$\begin{aligned} P_{H,t}C_t(j) + E_t\{\mathcal{F}_{t,t+1}^H Q_{H,t}(j)\} + \tilde{B}_{H,t}(j) + \mathcal{E}_t\tilde{B}_{F,t}(j) &\leq \\ &\leq \frac{1}{1 - \gamma} \left[ (1 + r_{t-1})\tilde{B}_{H,t-1}(j) + \mathcal{E}_t(1 + r_{t-1}^*)\tilde{B}_{F,t-1}(j) + Q_{H,t-1}(j) \right] + \\ &\quad + W_t N_t(j) + P_{H,t}D_t(j) - P_{H,t}T_t(j) \end{aligned} \quad (8)$$

where  $D_t(j) \equiv \int_0^1 D_t(h, j) dh$  denotes  $j$ 's claims on real profits from domestic firms,  $T_t(j)$  are real lump-sum taxes levied by the domestic fiscal authority on household  $j$ , and  $Q_{H,t}(j)$  denotes cohort  $j$ 's holdings of the portfolio of state-contingent assets, denominated in domestic currency, for which the relevant discount factor pricing one-period claims is  $\mathcal{F}_{t,t+1}^H$ .<sup>5</sup>

The solution of the optimization problem of domestic and foreign households delivers a set of cohort-specific equilibrium conditions which, once aggregated across cohorts, describe

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<sup>5</sup>An analogous budget constraint applies to foreign households. The stochastic discount factor is unique, within each country, given the assumption of complete domestic markets.

the aggregate labor supply and the dynamic path of aggregate consumption:<sup>6</sup>

$$\delta P_t C_t = W_t(1 - L_t), \quad (9)$$

$$C_t = \sigma E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} \right\} \Omega_t + \frac{1}{\beta} E_t \left\{ \mathcal{F}_{t,t+1} \frac{P_{t+1}}{P_t} C_{t+1} \right\}, \quad (10)$$

where  $W_t$  denotes the nominal wage and  $\Omega_t$  denotes the financial wealth in real terms.<sup>7</sup> The first term in (10) captures the financial-wealth effect on consumption, which is increasing in the turnover rate  $\gamma$ :

$$\sigma \equiv \gamma \frac{1 - \beta(1 - \gamma)}{\beta(1 - \gamma)}.$$

This additional term with respect to the representative-agent (RA) set up is a direct implication of the random replacement of a fraction of traders in the financial market with newcomers holding zero-wealth. Indeed, the interaction between long-time traders with accumulated wealth and newcomers holding zero financial wealth drives a wedge between the equilibrium stochastic discount factor and the *average* marginal rate of intertemporal substitution in consumption. In fact, while the cohort-specific Euler equation is the same as in the RA setup, given the insurance mechanism *à la* Blanchard, their aggregation is not straightforward (as it is in the RA setup) because the composition of traders in the financial markets tomorrow will include newcomers entering with zero-wealth to replace a share of old traders: the average consumption of these newcomers will therefore differ from that of old traders. Aggregation accounts for this difference by means of a wedge between the stochastic discount factor and the average marginal rate of substitution in consumption. Such wedge is proportional to the stock of financial wealth and creates a link between average consumption growth and the dynamics of financial wealth.

Notice that what drives the financial wealth effect is not the finiteness of individual agents' planning horizon, because the effect of this feature is sterilized by the insurance mechanism *à la* Blanchard. The financial wealth effect only appears in aggregate terms, and is truly implied by the presence of agents with zero-wealth and their interaction with old traders. This argument is crucial for the interpretation of the nature of parameter  $\gamma$ , and its possible quantitative calibration. As the rate of replacement ( $\gamma$ ) approaches zero the wealth effect

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<sup>6</sup>For details on the features of the model and the derivation of individual and aggregate equilibrium conditions, see Di Giorgio and Nisticò (2013).

<sup>7</sup>Specifically:

$$\Omega_{t-1}(j) \equiv \frac{1}{1 - \gamma} \frac{1}{P_t} \left[ (1 + r_{t-1}) \tilde{B}_{H,t-1}(j) + \mathcal{E}_t(1 + r_{t-1}^F) B_{F,t-1}^n(j) + Q_{H,t-1}(j) \right].$$

fades away and the model converges to the RA set up.<sup>8</sup>

## 2.2 The Government

In line with the related theoretical literature, and with empirical evidence, we assume that public consumption is fully home-biased: the government consumes an exogenously given amount of domestic goods only

$$G = \left[ \int_0^1 g(h)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}} \quad G^* = \left[ \int_0^1 g^*(f)^{\frac{\epsilon-1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}}. \quad (11)$$

Given Dixit-Stiglitz aggregation of the domestic public-consumption goods, public demand for brand  $h$  and  $f$  is equal to:

$$g(h) = \left( \frac{p(h)}{P_H} \right)^{-\epsilon} G \quad g^*(f) = \left( \frac{p^*(f)}{P_F^*} \right)^{-\epsilon} G^*. \quad (12)$$

Government consumption can be financed by levying lump-sum taxes  $T_t$  to domestic households or by issuing nominal debt denominated in local currency  $\tilde{B}_t$ . This implies the following flow budget constraint for the fiscal authority of country  $H$ , in nominal terms:<sup>9</sup>

$$\tilde{B}_t = (1 + r_{t-1})\tilde{B}_{t-1} + P_t Z_t, \quad (13)$$

where  $Z_t$  denotes the domestic real primary deficit, defined as

$$Z_t \equiv \frac{P_{H,t}}{P_t} G_t - T_t. \quad (14)$$

## 2.3 Market Clearing

Given the definition of terms of trade, the equilibrium relative prices follow:

$$\frac{P_H}{P} = \left[ \kappa + (1 - \kappa)\mathcal{S}^{1-\theta} \right]^{\frac{1}{\theta-1}} \equiv h(\mathcal{S}) \quad \frac{P_F^*}{P^*} = \left[ \kappa + (1 - \kappa)\mathcal{S}^{\theta-1} \right]^{\frac{1}{\theta-1}} \equiv f(\mathcal{S}),$$

which affect the equilibrium aggregate demands implied by market clearing in country  $H$

$$Y_H = \kappa \left( \frac{P_H}{P} \right)^{-\theta} C + (1 - \kappa) \left( \frac{P_H^*}{P^*} \right)^{-\theta} C^* + G \quad (15)$$

<sup>8</sup>For a discussion of this point, see Castelnovo and Nisticò (2010), and Nisticò (2014).

<sup>9</sup>A similar set of conditions holds for the foreign country.

and country  $F$

$$Y_F^* = (1 - \kappa) \left( \frac{P_F}{P} \right)^{-\theta} C + \kappa \left( \frac{P_F^*}{P^*} \right)^{-\theta} C^* + G^*. \quad (16)$$

For future reference, note that in the alternative case in which public spending is fully diversified between domestic and foreign goods, as in Obstfeld and Rogoff (1985), the aggregate demand equations read:

$$Y_H = \kappa \left( \frac{P_H}{P} \right)^{-\theta} C + (1 - \kappa) \left( \frac{P_H^*}{P^*} \right)^{-\theta} C^* + \left( \frac{P_H}{P_G} \right)^{-\theta} G^W \quad (17)$$

and

$$Y_F^* = (1 - \kappa) \left( \frac{P_F}{P} \right)^{-\theta} C + \kappa \left( \frac{P_F^*}{P^*} \right)^{-\theta} C^* + \left( \frac{P_F}{P_G^*} \right)^{-\theta} G^W, \quad (18)$$

in which  $G^W \equiv .5(G + G^*)$  and

$$\frac{P_H}{P_G} = \left[ .5 + .5\mathcal{S}^{1-\theta} \right]^{\frac{1}{\theta-1}} \quad \frac{P_F^*}{P_G^*} = \left[ .5 + .5\mathcal{S}^{\theta-1} \right]^{\frac{1}{\theta-1}},$$

with

$$P_G \equiv \left[ .5P_H^{1-\theta} + .5P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad P_G^* \equiv \left[ .5P_H^{*1-\theta} + .5P_F^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (19)$$

The Real Exchange Rate, finally, is determined by:

$$Q \equiv \frac{\mathcal{E}P^*}{P} = \left[ \frac{\kappa\mathcal{S}^{1-\theta} + (1 - \kappa)}{\kappa + (1 - \kappa)\mathcal{S}^{1-\theta}} \right]^{\frac{1}{1-\theta}}. \quad (20)$$

## 2.4 The Supply Side

We follow the insights of Basu and Kollmann (2013) and allow the government to potentially affect the private-sector productivity of labor by using a share  $\xi$  of total public spending to accumulate a stock of productive public capital  $\Gamma$  ( $\Gamma^*$  for the foreign country):

$$\Gamma_t = (1 - \eta)\Gamma_{t-1} + \xi G_t \quad \Gamma_t^* = (1 - \eta)\Gamma_{t-1}^* + \xi G_t^*,$$

where  $\eta$  is the rate of depreciation of public capital. In the steady state, the above law of motions imply

$$\bar{\Gamma} = \frac{\xi}{\eta} \bar{G} \quad \bar{\Gamma}^* = \frac{\xi}{\eta} \bar{G}^*.$$

Each firm producing brand  $h$  and  $f$  has therefore access to a linear technology:

$$Y_{H,t}(h) = \Gamma_t^\psi L_t(h), \quad Y_{F,t}^*(f) = \Gamma_t^{*\psi} L_t^*(f).$$

where  $\psi$  is the degree of public capital externality to labor productivity, and determines the steady-state marginal product of public capital (MPG):

$$MPG = \psi \frac{\bar{Y}_H}{\bar{\Gamma}} = \psi \frac{\eta \bar{Y}_H}{\xi \bar{G}} = \frac{\psi \eta}{\xi(1-\alpha)}, \quad (21)$$

and  $\alpha \equiv \frac{\bar{C}}{\bar{Y}_H}$  is the consumption-output ratio. We later use equation (21) to calibrate  $\psi$  consistently with empirical evidence.

Firms choose labor demand in a competitive labor market by minimizing their total real costs subject to the technological constraint. In equilibrium, the real marginal costs for the two countries are, respectively:

$$MC_t = \frac{P_t W_t}{P_{H,t} \Gamma_t^\psi} \quad MC_t^* = \frac{P_t^* W_t^*}{P_{F,t}^* \Gamma_t^{*\psi}}.$$

Using the brand-specific demand functions (6)–(7) and aggregating across domestic brands, we get the domestic aggregate production functions for the two countries:

$$Y_{H,t} \Xi_t = \Gamma_t^\psi L_t \quad Y_{F,t}^* \Xi_t^* = \Gamma_t^{*\psi} L_t^*,$$

in which

$$\Xi_t \equiv \int_0^1 \left( P_{H,t}(h) / P_{H,t} \right)^{-\epsilon} dh \quad \Xi_t^* \equiv \int_0^1 \left( P_{F,t}^*(f) / P_{F,t}^* \right)^{-\epsilon} df$$

captures (second-order) relative price dispersion among domestic firms in the two countries, and

$$L_t \equiv \int_0^1 L_t(h) dh \quad L_t^* \equiv \int_0^1 L_t^*(f) df$$

are the domestic and foreign aggregate per-capita amount of hours worked.

Equilibrium in the labor market then implies that real marginal costs equal

$$MC_t = \frac{\delta C_t}{\Gamma_t^\psi - Y_{H,t} \Xi_t} \frac{P_t}{P_{H,t}} \quad MC_t^* = \frac{\delta C_t^*}{\Gamma_t^{*\psi} - Y_{F,t}^* \Xi_t^*} \frac{P_t^*}{P_{F,t}^*} \quad (22)$$

for the domestic and foreign economy, respectively.

In each period, each firm faces a probability  $\vartheta$  of having to charge last period's price,

without re-optimizing. The problem of the firm is therefore to choose  $P_{H,t}^o$  in order to maximize

$$E_t \left\{ \sum_{k=0}^{\infty} \vartheta^k \mathcal{F}_{t,t+k} \left[ Y_{H,t+k|t}(h) P_{H,t}^o(h) - W_{t+k} P_{t+k} L_{t+k|t}(h) \right] \right\}$$

subject to

$$Y_{H,t+k|t}(h) = \left( \frac{P_{H,t}^o(h)}{P_{H,t+k}} \right)^{-\epsilon} Y_{H,t+k}$$

and

$$Y_{H,t+k|t}(h) = \Gamma_t^\psi L_{t+k|t}(h).$$

All firms re-optimizing at the same time will choose the same price, according to the following implicit rule:

$$E_t \left\{ \sum_{k=0}^{\infty} \vartheta^k \mathcal{F}_{t,t+k} Y_{H,t+k} P_{H,t+k}^\epsilon \left[ P_{H,t}^o - (1 + \mu) MC_{t+k} P_{H,t+k} \right] \right\} = 0.$$

Analogous implicit rules hold for the foreign country.

## 2.5 The Linear Model.

We analyze a first-order approximation of the model's equilibrium conditions around a zero-inflation/zero-deficit steady state.<sup>10</sup> Linearization around such steady state yields the complete set of linear equations needed to study the Rational-Expectation equilibrium (given stochastic processes for public spending).

Let lower-case variables denote percentage deviations from steady state  $x_t \equiv \frac{X_t - X}{X}$ .<sup>11</sup>

Our model economy can be summarized by the following linear equations. Nominal interest rates are linked through a standard Uncovered Interest-rate Parity (UIP) condition in real terms

$$r_t - E_t \pi_{t+1} = r_t^* - E_t \pi_{t+1}^* + E_t \Delta q_{t+1} \quad (23)$$

in which  $\pi_t \equiv \log(P_t/P_{t-1})$  and  $\pi_t^* \equiv \log(P_t^*/P_{t-1}^*)$  are the CPI-based inflation rate for

<sup>10</sup>Please refer to the Appendix for the complete non-linear model, the steady state and the flexible-price balanced-budget equilibrium (FBE).

<sup>11</sup>Except:  $c_t \equiv \frac{C_t - C}{Y_H}$ ,  $c_t^* \equiv \frac{C_t^* - C^*}{Y_F^*}$ ,  $g_t \equiv \frac{G_t - G}{Y_H}$ ,  $g_t^* \equiv \frac{G_t^* - G^*}{Y_F^*}$ ,  $\tau_t \equiv \frac{T_t - T}{Y_H}$ ,  $\tau_t^* \equiv \frac{T_t^* - T^*}{Y_F^*}$ ,  $\text{nfa}_t \equiv \frac{\text{NFA}_t}{Y_H}$ ,  $z_t \equiv \frac{Z_t}{Y_H}$ ,  $z_t^* \equiv \frac{Z_t^*}{Y_F^*}$ ,  $\text{nfa}_t^* \equiv \frac{\text{NFA}_t^*}{Y_H}$ ,  $b_t \equiv \frac{B_t}{Y_H}$ ,  $b_t^* \equiv \frac{B_t^*}{Y_F^*}$ ,  $\text{nx}_t \equiv \frac{\text{NX}_t}{Y_H}$ ,  $\omega_t \equiv \frac{\Omega_t}{Y_H}$ ,  $\gamma_t \equiv \frac{\Gamma_t - \Gamma}{Y_H}$  and  $\gamma_t^* \equiv \frac{\Gamma_t^* - \Gamma^*}{Y_F^*}$ .

country  $H$  and  $F$ , respectively:

$$\pi_t = \pi_{H,t} + (1 - \kappa)\Delta s_t \quad (24)$$

$$\pi_t^* = \pi_{F,t}^* - (1 - \kappa)\Delta s_t, \quad (25)$$

where the real exchange rate and terms of trade are related through

$$q_t = (2\kappa - 1)s_t. \quad (26)$$

Net foreign assets, expressed in terms of country  $H$ 's position, evolve as a function of consumption differential and the terms of trade:

$$\text{nfa}_t = \frac{1}{\beta}\text{nfa}_{t-1} + \frac{1}{2}(y_t^R - g_t^R - c_t^R) - \alpha(1 - \kappa)s_t \quad (27)$$

The equilibrium dynamics of aggregate consumption at Home and abroad follow

$$c_t = E_t c_{t+1} - \alpha(r_t - E_t \pi_{t+1} - \varrho) + \sigma \text{nfa}_t + \sigma b_t \quad (28)$$

$$c_t^* = E_t c_{t+1}^* - \alpha(r_t^* - E_t \pi_{t+1}^* - \varrho) - \sigma \text{nfa}_t + \sigma b_t^* \quad (29)$$

in which  $\varrho$  is the steady-state real interest rate. Public debt in real terms evolves according to

$$b_t = \beta^{-1}b_{t-1} + z_t \quad (30)$$

$$b_t^* = \beta^{-1}b_{t-1}^* + z_t^*, \quad (31)$$

where primary deficits are defined by

$$z_t = g_t - \tau_t - (1 - \alpha)(1 - \kappa)s_t \quad (32)$$

$$z_t^* = g_t^* - \tau_t^* + (1 - \alpha)(1 - \kappa)s_t. \quad (33)$$

On the supply side, Calvo price-setting implies two New Keynesian Phillips Curves of the usual kind:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda m c_{H,t} \quad \lambda \equiv \frac{(1 - \vartheta)(1 - \beta\vartheta)}{\vartheta} \quad (34)$$

$$\pi_{F,t}^* = \beta E_t \pi_{F,t+1}^* + \lambda m c_{F,t}^* \quad (35)$$

and the real equilibrium marginal costs follow:

$$mc_t = \frac{1}{\alpha} c_t + \varphi y_{H,t} + (1 - \kappa) s_t - \psi \frac{\eta(1 + \varphi)}{\xi(1 - \alpha)} \gamma_t \quad (36)$$

$$mc_t^* = \frac{1}{\alpha} c_t^* + \varphi y_{F,t}^* - (1 - \kappa) s_t - \psi \frac{\eta(1 + \varphi)}{\xi(1 - \alpha)} \gamma_t^* \quad (37)$$

where  $\varphi$  indicates the inverse of the steady-state Frisch elasticity of labor supply.

Under the assumed complete home bias in public spending, the total demand for domestic goods reads

$$y_{H,t} = 2\alpha\theta\kappa(1 - \kappa)s_t + \kappa c_t + (1 - \kappa)c_t^* + g_t \quad (38)$$

$$y_{F,t}^* = -2\alpha\theta\kappa(1 - \kappa)s_t + \kappa c_t^* + (1 - \kappa)c_t + g_t^* \quad (39)$$

In the case of fully diversified public spending (as assumed in Obstfeld and Rogoff, 1995, among others) equations (17) and (18) imply:

$$y_{H,t} = [2\alpha\theta\kappa(1 - \kappa) + 0.5\theta(1 - \alpha)]s_t + \kappa c_t + (1 - \kappa)c_t^* + g_t^W \quad (40)$$

$$y_{F,t}^* = -[2\alpha\theta\kappa(1 - \kappa) + 0.5\theta(1 - \alpha)]s_t + \kappa c_t^* + (1 - \kappa)c_t + g_t^W. \quad (41)$$

Finally, the law of motion of productive public capital are

$$\gamma_t = (1 - \eta)\gamma_{t-1} + \xi g_t \quad (42)$$

$$\gamma_t^* = (1 - \eta)\gamma_{t-1}^* + \xi g_t^*, \quad (43)$$

where public spending follows an AR(1) process ( $g_t = \rho_g g_{t-1} + u_{g,t}$ ) and relative quantities are defined as

$$c_t^R = c_t - c_t^* \quad (44)$$

$$y_t^R = y_{H,t} - y_{F,t}^* \quad (45)$$

$$g_t^R = g_t - g_t^* \quad (46)$$

The model is closed with two monetary rules and two fiscal rules. We assume in each country the presence of two policy makers: a Central Bank and a fiscal authority. The former sets the domestic nominal interest rate and the latter either public consumption or the level of domestic taxes.<sup>12</sup>

Monetary policy follows a simple instrument rule of the kind introduced by Taylor (1993),

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<sup>12</sup>In the following, we assume that the foreign authorities behave symmetrically.

where the nominal interest rate responds to deviations of the domestic inflation  $\pi_{H,t}$  and output gap  $x_t$  from the zero targets:

$$r_t = \varrho + \phi_\pi \pi_{H,t} + \phi_x x_t + u_{m,t}, \quad (47)$$

in which  $u_{m,t}$  are white noises capturing pure monetary policy shocks. As baseline calibration for the response coefficients and the volatility of monetary policy shocks, we use the following (symmetrized) values, consistent with the estimates provided for the U.S. and the Euro Area by Smets and Wouters (2003, 2007):  $\phi_\pi = \phi_\pi^* = 2$ ,  $\phi_x = \phi_x^* = 0.1$ ,  $\sigma_m = \sigma_m^* = 0.0016$ .

As to fiscal policy, we consider several alternative specifications, focusing only on “passive” (in the sense of Leeper, 1991) or implementable (in the sense of Schmitt-Grohe and Uribe, 2007) fiscal rules. The first specification considers the case in which the government targets a balanced budget in every period:

$$z_t = 0. \quad (48)$$

In this case, an increase in public consumption is financed through an equivalent increase in domestic taxes.

Given the non-Ricardian structure of our model, we can also analyze alternative fiscal regimes which do not imply a balanced budget in every period. In particular, one alternative regime has real taxes follow an exogenous, stationary autoregressive process:

$$\tau_t = \rho_\tau \tau_{t-1} + (1 - \rho_\tau) \xi_b b_{t-1} + u_{z,t}, \quad (49)$$

where a drift adjusting to the stock of outstanding debt insures fiscal solvency ( $\xi_b = \varrho/s_c$ ). In this regime, therefore, an increase in public consumption is financed through new debt.

The third specification considers the case in which governments set their primary deficit following a counter-cyclical feedback rule of the kind:

$$z_t = -\mu_b b_{t-1} - \mu_x x_t + u_{z,t}. \quad (50)$$

This specification was analyzed by recent empirical and theoretical literature (see Galí and Perotti, 2003 and Di Giorgio and Nisticò, 2013), and encompasses different fiscal regimes, depending on the specific values for the response coefficients.

If the response coefficients on the output gap are zero and those on the stock of debt as low as needed to ensure determinacy, this fiscal rule corresponds to fiscal regime (49), and, therefore, an increase in public consumption is simply and entirely financed through new debt. Non-zero response coefficients, on the other hand, imply that the fiscal regime responds to the business cycle and the dynamics of the public debt, potentially affecting the transmission

mechanism of any kind of shock. In this scenario, we calibrate the response coefficients with the following (symmetrized) values, consistent with the estimates provided by Galí and Perotti (2003) for the U.S. and the group of EMU10:  $\mu_x = \mu_x^* = 0.7$ ,  $\mu_b = \mu_b^* = 0.01$ .

## 2.6 Parameterization

We parameterize the structural model on a quarterly frequency, following previous studies and convention, and consistently with Di Giorgio and Nisticò (2013). Specifically, the steady-state net quarterly interest rate  $\rho$  was set at 0.01.<sup>13</sup> The rate of replacement  $\gamma$  was set equal to 0.1, consistently with the evidence for the U.S. recently provided, in a related framework, by Castelnuovo and Nisticò (2010). In order to meet the steady-state restrictions, the intertemporal discount factor  $\beta$  was set at 0.99. The degree of monopolistic competition is taken from Rotemberg and Woodford (1997),  $\epsilon = 7.66$ , which implies an average markup of 15%. In line with estimates provided for the U.S. by Smets and Wouters (2007), we set the Calvo parameter at 0.75, implying that prices are revised on average once a year. As to the elasticity of real wages to aggregate hours  $\varphi$ , we choose a baseline value of 0.5, consistently with Rotemberg and Woodford (1997) and implying 8 hours worked in steady-state. The elasticity of substitution between Home and Foreign goods was set equal to  $\theta = 1.5$ , which implies that home and foreign goods are substitute in the utility function of consumers. Finally, the degree of home bias in private consumption is parametrized by assuming  $\kappa = 0.6$ .

With respect to the calibration of the supply side, we set  $\alpha = 0.8$  to match consumption-output ratio,  $\eta/\xi = 0.1$  to match a public capital-to-output ratio of  $\bar{\Gamma}/\bar{Y}_H = \frac{\xi}{\eta}(1 - \alpha) = 2$ , consistently with Kamps (2006), that reports for most OECD countries a ratio of about 0.5 in annual terms. Based on equation (21), we set  $\psi = 0.6$  to match the MPG of 0.3, in line with the evidence provided by Bom and Ligthart (2009) that report short-run MPG in the range between .08 and .34 and long-run MPG in the range between .32 and .48. Finally, we set  $\xi = 0.4$ , consistently with the ratio  $\frac{EDU+GFI}{GC+GFI}$  for the U.S. in the Great-Moderation sample, where  $GC$  is government consumption,  $GFI$  is public gross fixed investment and  $EDU$  is government spending in education, the latter proxying for public investment in human capital. Finally, to calibrate persistence and volatility of the fiscal shocks, we estimate an independent AR(1) process for each shock, using quarterly HP-filtered data on government consumption and real personal taxes in the U.S. and the Euro Area for the available sample (1970:1 to 2005:4). The values obtained are reported in Table 1. Given the structural symmetry of our framework, we follow Backus, Kehoe and Kydland (1992), among the others, and use for

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<sup>13</sup>Since we focus on a symmetric steady state the values reported in the text are meant to refer to both countries as well as to the world economy.

**Table 1:** Stochastic properties of the fiscal shocks.

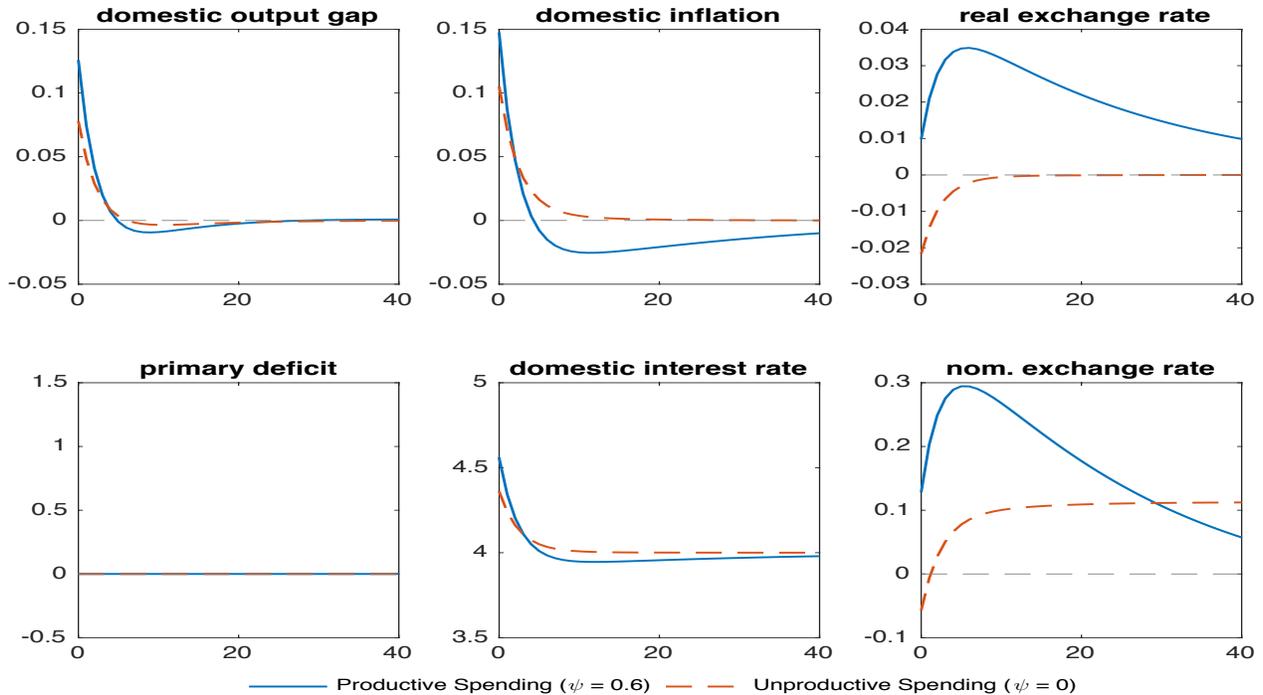
Shock	$\rho_g$	$\sigma_g$	Adj. $R^2$	Shock	$\rho_\tau$	$\sigma_\tau$	Adj. $R^2$
$g$	<b>0.692</b> (11.164)	<b>0.0066</b>	<b>0.4674</b>	$\tau$	<b>0.768</b> (14.008)	<b>0.0192</b>	<b>0.5802</b>
$g^*$	<b>0.638</b> (10.056)	<b>0.0041</b>	<b>0.4159</b>	$\tau^*$	<b>0.905</b> (25.269)	<b>0.0105</b>	<b>0.8181</b>

the benchmark simulation a symmetrized version of our estimates. We therefore calibrate  $\rho_g = \rho_g^* = 0.665$ ,  $\sigma_g = \sigma_g^* = 0.0054$ ,  $\rho_\tau = \rho_\tau^* = 0.836$  and  $\sigma_\tau = \sigma_\tau^* = 0.0148$ .

### 3 Fiscal Policy

This Section evaluates the dynamic effects of a wide range of fiscal policy shocks, and compares their real-exchange-rate implications with those found in the literature. Figure 1 shows the effects of a balanced-budget increase in public spending in the model with productive public capital (solid line) and in the benchmark case where public spending is unproductive, which is nested in our model when  $\psi = 0$  (dashed line). If government spending is productive, fiscal shocks affect marginal costs and inflation through two channels. They imply the familiar inflationary pressures through higher aggregate demand (“demand-side channel”); but they also imply a downward pressure on both marginal costs and inflation given the higher productivity induced in the private sector (“supply-side channel”). If the supply-side (demand-side) channel is stronger, domestic inflation decreases (increases), implying relatively lower (higher) domestic interest rates and an exchange rate depreciation (appreciation). The solid line in figure 1 shows that after a positive shock in government spending domestic inflation rises on impact, but in the medium term, through a reduction in marginal costs, it falls below its steady-state level, triggering a relative reduction in the nominal interest rate and, thereby, an exchange rate depreciation, both nominal and real. The law of motion of public capital, moreover, implies additional persistency in the propagation of the shock along the “supply-side channel”, thereby inducing a hump-shaped response of both the nominal and real exchange rate which is consistent with several empirical studies. In the benchmark model, instead, the real exchange rate appreciates on impact, as the fiscal shock propagates only through the “demand-side channel”: relative interest rates increase to offset the inflationary pressures and induce an appreciation of the real exchange rate.

Di Giorgio et. al. (2015) study a generalized version of the Obstfeld and Rogoff (1995)



**Figure 1:** Response of selected variables to a 1%, balanced-budget increase in public spending. Solid line: model with productive public spending; dashed line: standard NOEM model with unproductive public spending.

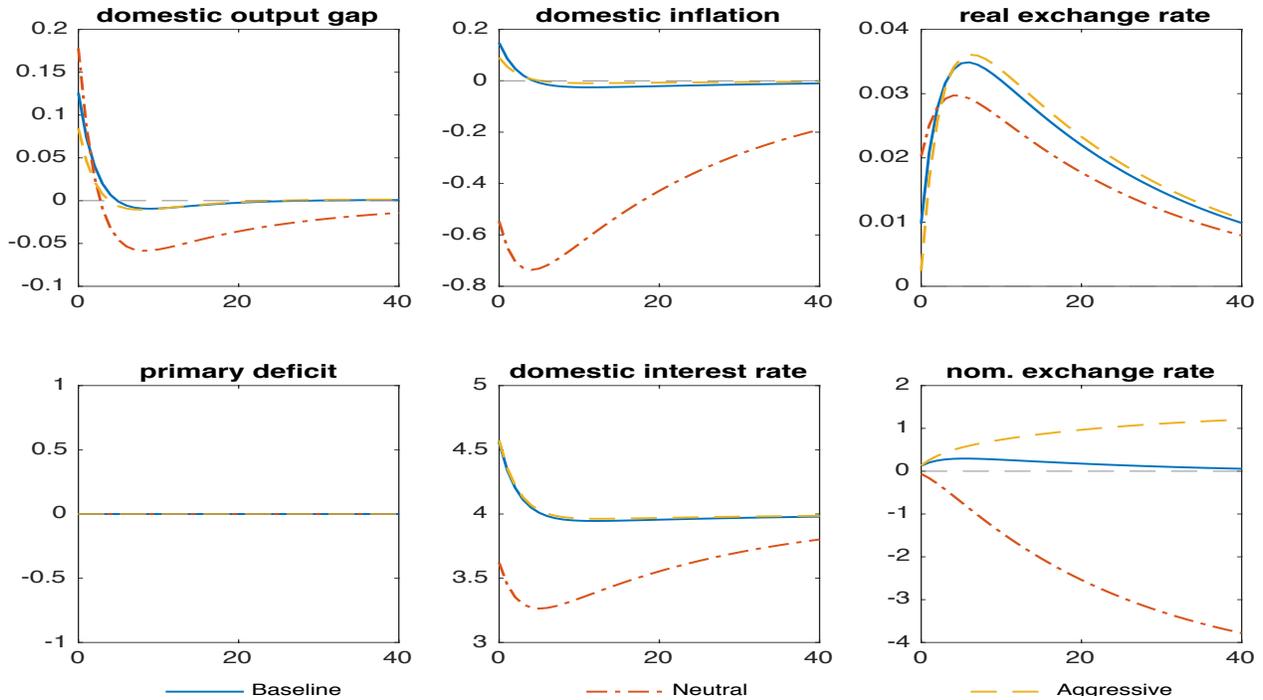
well known Redux model (where fiscal expansions yield an exchange-rate depreciation) and show how different structural assumptions can impact on the exchange rate response. In particular, they investigate the role of endogenous monetary policy and home bias in public consumption and find that a counterfactual appreciation of the exchange rate may be obtained. Here we evaluate the relevance of these two features for an economy with productive public spending.

We first study the role of endogenous monetary policy. In figure 2 we vary the feedback response to output and inflation and compare three alternative degrees of endogeneity of the monetary policy rule (47): the baseline specification of Figure 1 ( $\phi_\pi = 2$ ,  $\phi_x = 0.1$ ), a “neutral” specification with a zero response to output ( $\phi_x = 0$ ) and a minimal response to inflation ( $\phi_\pi = 1$ ), and finally an “aggressive” specification where the response coefficients are doubled with respect to the baseline case.<sup>14</sup>

In the standard NOEM model with unproductive public spending, stronger feedback responses to output and inflation induce stronger appreciations in both the nominal and the real exchange rate.<sup>15</sup> In contrast, Figure 2 shows that if public spending is productive, a fiscal expansion induces a persistent depreciation of the real exchange rate regardless of

<sup>14</sup>In all cases, the policy regime is symmetric across countries.

<sup>15</sup>Figures are available on request.

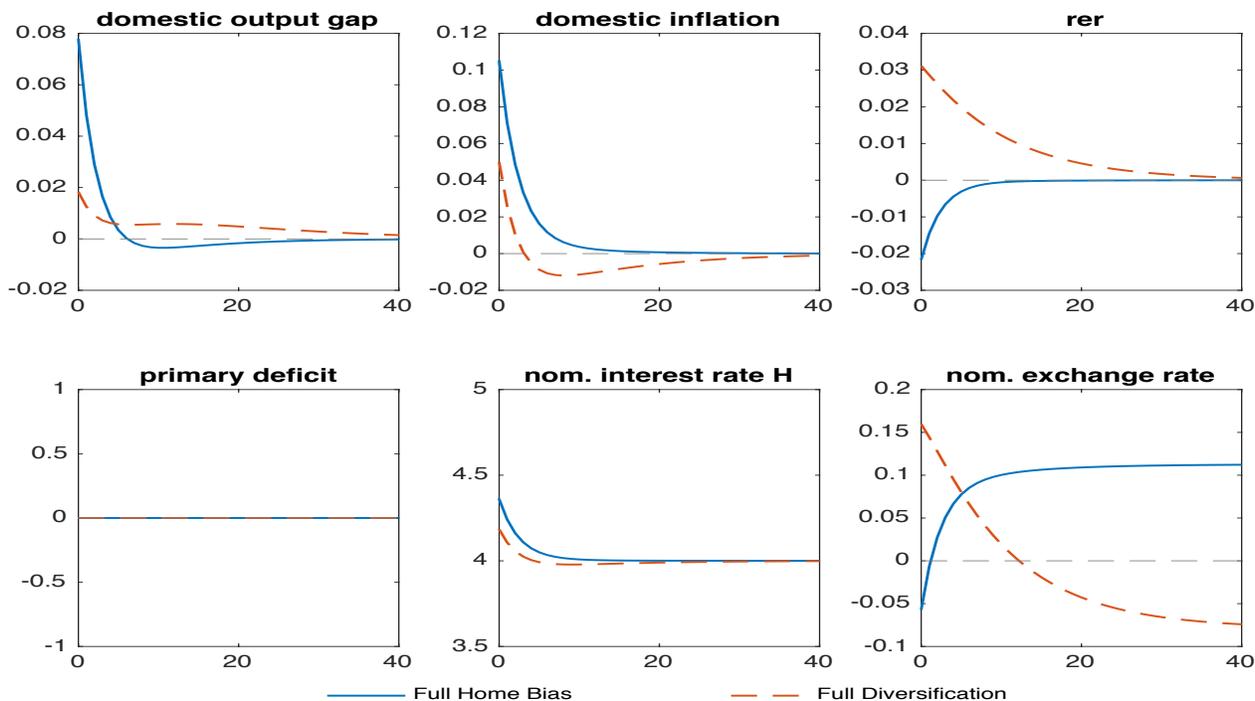


**Figure 2:** Balanced-budget increase in public spending: the role of endogenous monetary policy. Solid line: “baseline” case; dashed-dotted line: “neutral” case; dashed line: “aggressive” case.

monetary policy. On the other hand, the qualitative response of the nominal exchange rate is not invariant: a “neutral” specification of the Taylor Rule, indeed, implies a fall in relative nominal interest rates and, through the UIP, a substantial appreciation of the nominal exchange rate.<sup>16</sup>

In Figure 3 we investigate the role played by the composition of public consumption in the standard NOEM model. As emphasized in Ganelli (2005a) and Di Giorgio et al. (2015), the degree of home bias in public spending is another key variable driving the exchange rate response to fiscal shocks. Figure 3 shows that diversifying public spending across domestic and foreign goods implies a real exchange rate depreciation, as in Obstfeld and Rogoff (1995). Although the result is the same as in the Redux model, however, the transmission mechanism is radically different. In Obstfeld and Rogoff (1995) an increase in public consumption crowds out consumption both at home and abroad; however, since monetary policy is exogenous and the fiscal expansion is financed by an increase of domestic taxes only, domestic consumption falls more than foreign one, and the ensuing excess supply of money is higher at home than abroad. The exchange rate therefore depreciates. In our New-Keynesian model, instead, the transmission mechanism works through marginal costs and the endogenous monetary policy

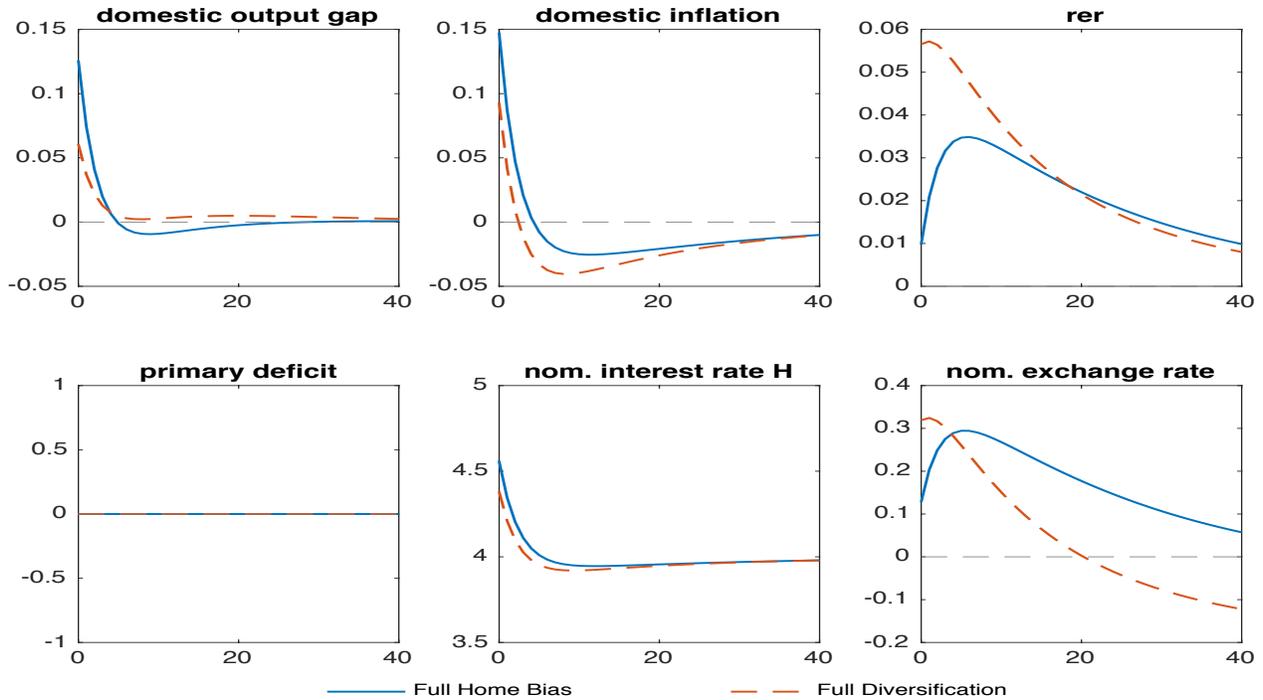
<sup>16</sup>In a separate simulation, available on demand, we show that a monetary policy regime pegging the real interest rate in both countries implies that an increase in productive government spending has clearly a null effect on the real exchange rate – through equation (23) – while appreciating the nominal exchange rate.



**Figure 3:** Balanced-budget increase in public spending: the role of home bias in public spending in the benchmark NOEM model. Solid line: full home bias; dashed line: full diversification.

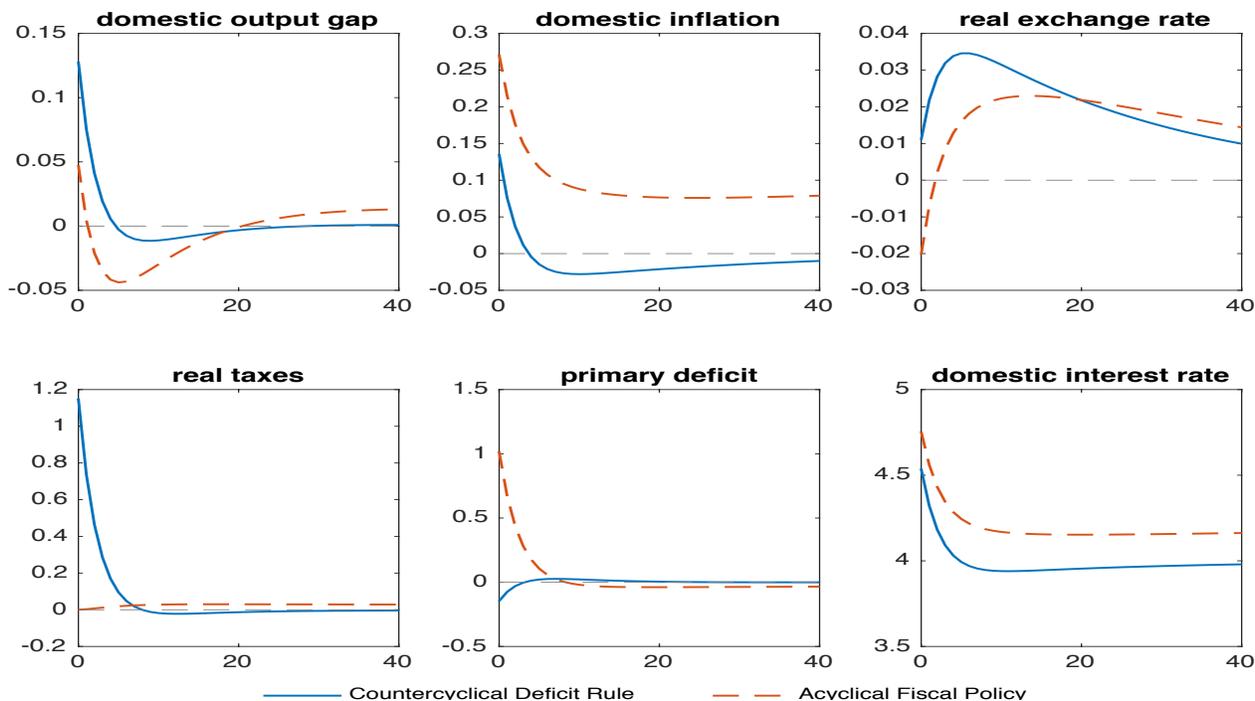
reaction. An increase in public spending that is directed towards both home and foreign goods has positive effects on the marginal costs of both countries. However, since the fiscal expansion is financed by an increase in domestic taxes only, relative consumption falls. As a result, foreign marginal costs and inflation increase more than domestic ones, triggering a relatively stronger response by foreign monetary policy. Contrary to the case of home-biased public spending (solid line in figure 3), therefore, the relative interest rate falls and the nominal and the real exchange rate depreciate.

Figure 4 extends this exercise to our model with productive government spending. The real exchange rate depreciates regardless of the specific assumption on the degree of home bias in public consumption. Notice that, in the case of full diversification (dashed line), the increase in public spending increases marginal costs in both countries; however, since the fiscal expansion only affects domestic labor productivity, relative marginal costs and inflation are lower, triggering a fall in relative interest rates and a nominal and real depreciation. In this case, the demand effect on domestic inflation is smaller in size since part of this expansion falls on foreign goods. As a consequence, the on-impact depreciation is higher than in the case of complete home bias, while the long run path is similar. While under the assumption of home-biased public spending the nominal exchange rate remains depreciated throughout the transition, in the case of full international diversification, it gradually converges to a permanently appreciated level.



**Figure 4:** Balanced-budget increase in public spending: the role of home bias in public spending in the model with productive public spending. Solid line: full home bias; dashed line: full diversification.

All the effects discussed so far are clearly independent of the overlapping-generation structure of our DNK model. The balanced-budget specification of the fiscal expansions considered thus far does not allow to consider the accumulation of public debt and therefore the role of wealth effects. Our overlapping-generation structure, however, allows us to go further and evaluate whether the results obtained in the case of balanced-budget spending shocks are robust to alternative assumptions about the financing of the latter. Figure 5 shows the effects of an unbalanced increase in government spending under two different fiscal regimes: an endogenous countercyclical fiscal policy (solid line), described by rule (50), and exogenous fiscal policy (dashed line), where real taxes follow a stationary autoregressive process such as (49). The simulation shows how the fiscal policy regime affects the on impact reaction of the real exchange rate. The exchange rate depreciates under endogenous fiscal policy (more intensively with respect to the balanced budget case displayed in Figure 1). Under an exogenous tax rule, the government runs a significant primary deficit which amplifies the demand effect of the fiscal shock. As a consequence, domestic inflation is much higher and the real exchange rate appreciates on impact. Notice, however, that after the short-run appreciation, the real exchange rate overshoots its long-run equilibrium level and remains depreciated throughout the transition, as in the case of endogenous fiscal policy. In the medium run, moreover, the real exchange rate depreciates even more when fiscal policy is exogenous, given the accumulation of net foreign liabilities implied by the lack of fiscal discipline. The degree

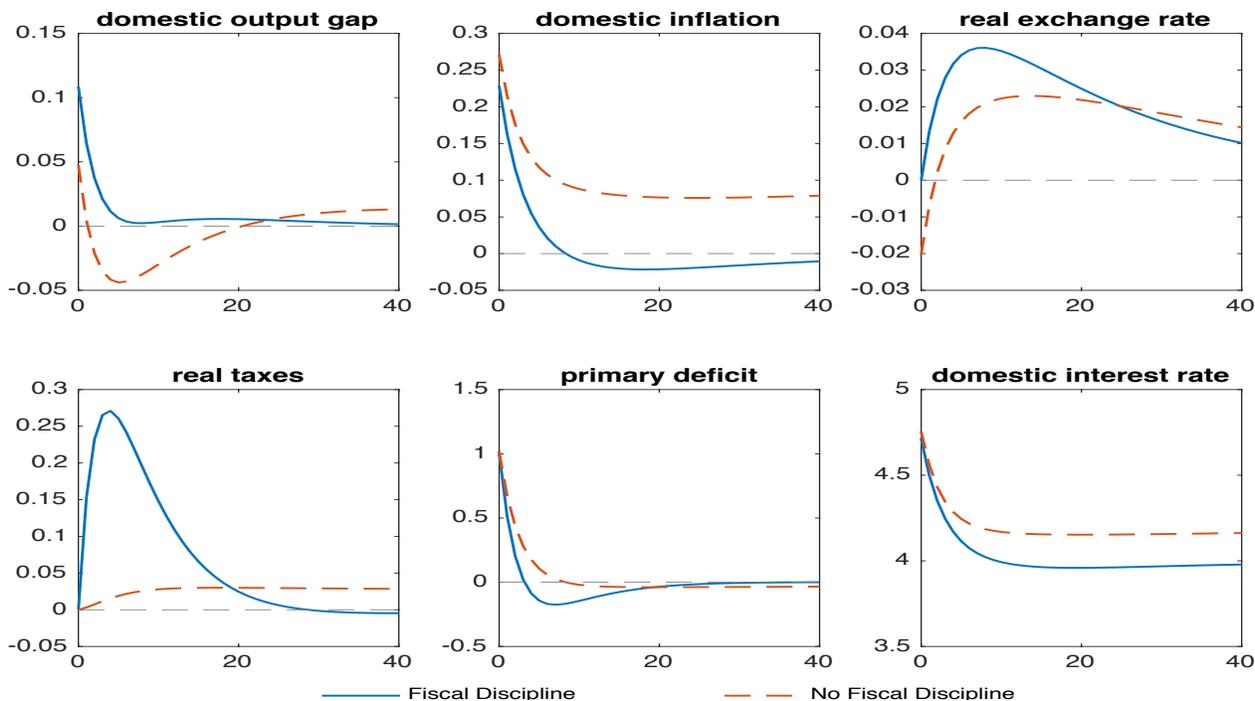


**Figure 5:** Non balanced-budget increase in public spending: the role of endogenous countercyclical fiscal rules. Solid line: endogenous countercyclical fiscal policy; Dashed line: exogenous fiscal policy.

of fiscal discipline is another non-negligible dimension affecting the exchange-rate response to public spending shocks. Figure 6 compares the exogenous fiscal regime of Figure 5 (dashed line) with one in which the tax rule responds more aggressively to the level of public debt ( $\xi_b = 0.1$  in rule (49)): the real exchange rate depreciates also on impact, the more so the more disciplined is fiscal policy towards public debt (see solid line in Figure 6).<sup>17</sup>

The previous investigation allows us to better understand the effects on the exchange rate of different ways of financing public spending. In particular, the effects of a debt-financed increase in public spending can be decomposed as the sum of a tax cut and a balanced-budget expansion. Using a perpetual-youth version of the *Redux* model, Ganelli (2005b) found that, while a balanced-budget expansion would imply a depreciation through a reduction in relative consumption and an increase in domestic prices, a tax-cut tends to appreciate the exchange rate on impact. As a consequence, the final effect on the exchange rate is ambiguous. Our simulations overcome the ambiguity result of Ganelli (2005b). In our framework with productive public capital, a debt-financed expansion in public consumption unambiguously induces a real depreciation in the medium run. The real exchange rate depreciates on impact

<sup>17</sup>We also simulated the effect on an expansionary fiscal policy conducted via a debt-financed tax cut, under alternative degrees of fiscal discipline. Clearly, in this case the expansionary effect of fiscal shock is transmitted only through the “demand-side channel”. Since it does not affect labor productivity, the tax cut triggers a persistent exchange rate appreciation as in the standard NOEM models without public capital. The result holds independently of the specific degree of fiscal discipline.



**Figure 6:** Debt-financed increase in public spending: the role of fiscal discipline. Solid line: exogenous fiscal policy with fiscal discipline; Dashed line: exogenous fiscal policy with no fiscal discipline.

when fiscal policy is “sufficiently” aggressive, as shown by Figure 5 and 6.

Overall, the results of this section suggest that the role played by government-spending externality on labor productivity allows to reconcile the prediction of a general NOEM model with the widespread evidence documenting a real exchange rate depreciation following an expansionary fiscal policy.

## 4 Concluding Remarks

This paper provides a characterization of the effects of different fiscal shocks on the exchange rate. We analyze both balanced-budget and debt financed fiscal expansions when government spending positively affects labor productivity in the private sector. Consistently with empirical evidence, the real exchange rate generally depreciates following an increase in public spending – both on impact and in the transition – while in standard NOEM models the same shock produces an exchange-rate appreciation. The mechanism we analyze is able to offset the positive effect on marginal costs induced by higher domestic demand following an increase in government purchases. In our model, a positive shock in public spending improves private sector’s productivity: this lowers domestic real marginal costs and triggers a lower interest rate response by the domestic monetary authority. Consequently, the real exchange

rate depreciates.

We also show that the short-run response of the exchange rate is, in general, affected by public spending composition, the degree of endogeneity in monetary policy, and also the overall fiscal regime (in terms of the degrees of counter-cyclicality and fiscal discipline) when the spending shock is unbalanced. In particular, in the latter case, the real exchange rate depreciates on impact if the government follows either a sufficiently countercyclical feedback rule on the primary deficit or a tax rule with sufficiently high degree of fiscal discipline. In all cases, however, we find a persistent depreciation in the medium run.

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# Appendix

## A The Complete Model

At time  $t$ , the complete set of conditions needed to study the international equilibrium are:

1. labor-supply schedules, both countries

$$\delta C_t = W_t(1 - L_t) \quad (51)$$

$$\delta C_t^* = W_t^*(1 - L_t^*) \quad (52)$$

2. private-sector budget constraints, aggregate per-capita terms, both countries

$$C_t + E_t \mathcal{F}_{t,t+1} \Omega_t = W_t L_t + D_t - T_t + \frac{\Omega_{t-1}}{1 + \pi_t} \quad (53)$$

$$C_t^* + E_t \mathcal{F}_{t,t+1}^* \Omega_t^* = W_t^* L_t^* + D_t^* - T_t^* + \frac{\Omega_{t-1}^*}{1 + \pi_t^*} \quad (54)$$

3. dynamic equations for aggregate consumption, both countries

$$C_t = \sigma E_t \mathcal{F}_{t,t+1} \Omega_t + \frac{1}{\beta} E_t \{ \mathcal{F}_{t,t+1} (1 + \pi_{t+1}) C_{t+1} \} \quad (55)$$

$$C_t^* = \sigma E_t \mathcal{F}_{t,t+1}^* \Omega_t^* + \frac{1}{\beta} E_t \{ \mathcal{F}_{t,t+1}^* (1 + \pi_{t+1}^*) C_{t+1}^* \} \quad (56)$$

4. relative prices, both countries

$$\frac{P_{H,t}}{P_t} = \left[ \kappa + (1 - \kappa) \mathcal{S}_t^{1-\theta} \right]^{\frac{1}{\theta-1}} \equiv h(\mathcal{S}_t) \quad (57)$$

$$\frac{P_{F,t}^*}{P_t^*} = \left[ \kappa + (1 - \kappa) \mathcal{S}_t^{\theta-1} \right]^{\frac{1}{\theta-1}} \equiv f(\mathcal{S}_t) \quad (58)$$

Where  $\mathcal{S}_t$  denotes the terms of trade:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{P_{F,t}^*}{P_{H,t}^*}$$

5. demand for domestic goods, both sectors and both countries

$$Y_{H,t} = \kappa \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C_t + (1 - \kappa) \mathcal{S}_t^\theta \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^* + G_t \quad (59)$$

$$Y_{F,t}^* = (1 - \kappa) \mathcal{S}_t^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C_t + \kappa \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^* + G_t^* \quad (60)$$

6. public-sector budget constraints, per-capita terms, both countries

$$B_t = \frac{1 + r_{t-1}}{1 + \pi_t} B_{t-1} + Z_t, \quad (61)$$

$$B_t^* = \frac{1 + r_{t-1}^*}{1 + \pi_t^*} B_{t-1}^* + Z_t^*, \quad (62)$$

$$Z_t = \frac{P_{H,t}}{P_t} G_t - T_t \quad (63)$$

$$Z_t^* = \frac{P_{F,t}^*}{P_t^*} G_t^* - T_t^* \quad (64)$$

7. real marginal costs, both countries

$$MC_t = \frac{W_t}{\Gamma_t^\psi} \frac{P_t}{P_{H,t}} \quad (65)$$

$$MC_t^* = \frac{W_t^*}{\Gamma_t^{*\psi}} \frac{P_t^*}{P_{F,t}^*} \quad (66)$$

8. aggregate production functions, both sectors and both countries

$$\Xi_t Y_{H,t} = \Gamma_t^\psi L_t \quad (67)$$

$$\Xi_t^* Y_{F,t}^* = \Gamma_t^{*\psi} L_t^* \quad (68)$$

9. aggregate dividends, economy-wide, both countries

$$D_t = \frac{P_{H,t}}{P_t} Y_{H,t} - W_t L_t \quad (69)$$

$$D_t^* = \frac{P_{F,t}^*}{P_t^*} Y_{F,t}^* - W_t^* L_t^* \quad (70)$$

10. Uncovered Interest Parity

$$E_t \left\{ \mathcal{F}_{t,t+1} \left[ \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} (1 + r_t^*) - (1 + r_t) \right] \right\} = 0 \quad (71)$$

11. optimal consumer-price setting equations, both countries

$$E_t \left\{ \sum_{k=0}^{\infty} \vartheta^k \mathcal{F}_{t,t+k} Y_{H,t+k} P_{H,t+k}^\epsilon \left[ P_{H,t}^o - (1 + \mu) MC_{t+k} P_{H,t+k} \right] \right\} = 0 \quad (72)$$

$$E_t \left\{ \sum_{k=0}^{\infty} \vartheta^k \mathcal{F}_{t,t+k}^* Y_{F,t+k}^* P_{F,t+k}^{*\epsilon} \left[ P_{F,t}^{*o} - (1 + \mu) MC_{t+k}^* P_{F,t+k}^* \right] \right\} = 0 \quad (73)$$

Moreover, the following definitions hold:

$$\text{NFA}_t \equiv \Omega_t - B_t = -Q_t \text{NFA}_t^* \quad Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{h(\mathcal{S}_t)}{f(\mathcal{S}_t)} \mathcal{S}_t$$

$$\text{CA}_t \equiv \text{NFA}_t - \frac{\text{NFA}_{t-1}}{1 + \pi_t}$$

## B The Steady-State.

In this Section we derive the relations characterizing a zero-inflation, zero-deficit symmetric steady state. In such steady state we have, by assumption,  $\bar{G} = \bar{G}^*$  and  $\bar{Z} = \bar{Z}^* = 0$ . Moreover,  $\bar{MC} = \bar{MC}^* = (1 + \mu)^{-1}$  and  $\bar{\Gamma} = \bar{\Gamma}^*$ .

This further implies  $\bar{B} = \bar{B}^* = \bar{\Omega} = \bar{\Omega}^* = 0$  and  $\bar{S} = \bar{Q} = 1$ . The Uncovered Interest Parity implies  $r = r^*$ . The latter, together with equations (9)–(10), implies  $r = (1 - \beta)/\beta$  and  $\bar{C} = \bar{C}^*$ .

Equations (59) and (60) imply:

$$\bar{C} + \bar{G} = \bar{Y}_H = \bar{Y}_F^*.$$

Equilibrium marginal costs, labor supplies and production functions in each country further imply (given structural symmetry):

$$\begin{aligned}\bar{Y}_H &= \frac{\bar{\Gamma}^\psi + \delta(1 + \mu)\bar{G}}{1 + \delta(1 + \mu)} \\ \bar{C} &= \frac{\bar{\Gamma}^\psi - \bar{G}}{1 + \delta(1 + \mu)} \\ \bar{L} &= \frac{1 + \delta(1 + \mu)\bar{G}\bar{\Gamma}^{-\psi}}{1 + \delta(1 + \mu)} \\ \bar{W} &= \frac{\bar{\Gamma}^\psi}{1 + \mu}.\end{aligned}$$

Notice that the non-negativity constraint on consumption implies the following restriction on the parameters governing the dynamics of public capital:

$$\bar{\Gamma}^\psi > \bar{G}.$$

From the above, the following two composite parameters derive

$$\varphi \equiv \frac{\bar{L}}{1 - \bar{L}} = \frac{1 + \delta(1 + \mu)\bar{G}\bar{\Gamma}^{-\psi}}{\delta(1 + \mu)(1 - \bar{G}\bar{\Gamma}^{-\psi})} \quad (74)$$

$$\alpha \equiv \frac{\bar{C}}{\bar{Y}_H} = \frac{\bar{C}^*}{\bar{Y}_F^*} = \frac{1 - \bar{G}\bar{\Gamma}^{-\psi}}{1 + \delta(1 + \mu)\bar{G}\bar{\Gamma}^{-\psi}}, \quad (75)$$

which are related to each other through

$$1 = \alpha\delta(1 + \mu)\varphi. \quad (76)$$

## C The Flexible-Price Balanced-Budget Equilibrium (FBE)

Following Di Giorgio and Nisticò (2013), we take as our benchmark the global allocation arising in the absence of all distortions: the Flexible-price Balanced-budget Equilibrium (FBE). The “flexible-price” feature of the FBE rules out the distortions due to monopolistic competition and nominal rigidities, to a first-order approximation. The “zero-deficit” (balanced-budget) feature instead sterilizes – at the world level – the third distortion due to the consumption dispersion across agents: since it implies a zero-stock of global financial wealth, the dynamics of global consumption is undistorted. In the Flexible-price Balanced-budget Equilibrium, therefore, the global fluctuations induced by structural shocks reflect the efficient dynamic response of the economy.

In the FBE, marginal costs in both countries are equal to their steady state level at all times. Let  $\bar{m}_t$  denote the level of generic variable  $m_t$  in the FBE at time  $t$ , the relevant system of linear equations therefore becomes:

$$\bar{q}_t = (2\kappa - 1)\bar{s}_t \quad (77)$$

$$0 = \frac{1}{\alpha}\bar{c}_t^R + \varphi\bar{y}_t^R + 2(1 - \kappa)\bar{s}_t - \psi \frac{\eta(1 + \varphi)}{\xi(1 - \alpha)}(\gamma_t - \gamma_t^*) \quad (78)$$

$$\bar{y}_{H,t} = 2\alpha\theta\kappa(1 - \kappa)\bar{s}_t + \kappa\bar{c}_t + (1 - \kappa)\bar{c}_t^* + g_t \quad (79)$$

$$\bar{y}_{F,t}^* = -2\alpha\theta\kappa(1 - \kappa)\bar{s}_t + \kappa\bar{c}_t^* + (1 - \kappa)\bar{c}_t + g_t^* \quad (80)$$

$$0 = \frac{1}{\alpha}(\bar{c}_t + \bar{c}_t^*) + \varphi(\bar{y}_{H,t} + \bar{y}_{F,t}^*) - \psi \frac{\eta(1 + \varphi)}{\xi(1 - \alpha)}(\gamma_t + \gamma_t^*) \quad (81)$$

$$\bar{c}_t^R = E_t\bar{c}_{t+1}^R - \alpha E_t\Delta\bar{q}_{t+1} + 2\sigma\overline{\text{nfa}}_t + \sigma\bar{b}_t^R \quad (82)$$

$$\overline{\text{nfa}}_t = \frac{1}{\beta}\overline{\text{nfa}}_{t-1} + \frac{1}{2}(\bar{y}_t^R - g_t^R - \bar{c}_t^R) - \alpha(1 - \kappa)\bar{s}_t \quad (83)$$

$$\bar{b}_t = \beta^{-1}\bar{b}_{t-1} + \bar{z}_t \quad (84)$$

$$\bar{b}_t^* = \beta^{-1}\bar{b}_{t-1}^* + \bar{z}_t^* \quad (85)$$

$$\bar{z}_t = g_t - \bar{\tau}_t - (1 - \alpha)(1 - \kappa)\bar{s}_t \quad (86)$$

$$\bar{z}_t^* = g_t^* - \bar{\tau}_t^* + (1 - \alpha)(1 - \kappa)\bar{s}_t \quad (87)$$

$$\gamma_t = (1 - \eta)\gamma_{t-1} + \xi g_t \quad (88)$$

$$\gamma_t^* = (1 - \eta)\gamma_{t-1}^* + \xi g_t^* \quad (89)$$

$$\bar{y}_t^R = \bar{y}_t - \bar{y}_t^* \quad (90)$$

$$\bar{c}_t^R = \bar{c}_t - \bar{c}_t^* \quad (91)$$

$$\bar{g}_t^R = \bar{g}_t - \bar{g}_t^* \quad (92)$$

$$\bar{b}_t^R = \bar{b}_t - \bar{b}_t^* \quad (93)$$

$$\bar{z}_t = \frac{\beta - 1}{\beta}\bar{b}_{t-1} \quad (94)$$

$$\bar{z}_t^* = \frac{\beta - 1}{\beta}\bar{b}_{t-1}^* \quad (95)$$

The last two equations in the system above define the fiscal rules that implement the FBE and guarantee that the levels of home and foreign real debt are stabilized at their initial levels:  $\bar{b}_{-1} = 0$  and  $\bar{b}_{-1}^* = 0$ . Specifying some initial conditions also for the net foreign asset position and relative prices ( $\overline{\text{nfa}}_{-1} = 0$ , and  $\bar{s}_{-1} = 0$ ), the above system of 19 stochastic difference equations delivers as a solution the equilibrium values of the 19 endogenous variables:

$$\{\gamma_t, \gamma_t^*, \bar{q}_t, \bar{s}_t, \overline{\text{nfa}}_t, \bar{b}_t, \bar{b}_t^*, \bar{c}_t, \bar{c}_t^*, \bar{c}_t^R, \bar{g}_t, \bar{g}_t^R, \bar{y}_{H,t}, \bar{y}_{F,t}^*, \bar{y}_t^R, \bar{b}_t^R, \bar{z}_t, \bar{z}_t^*, \bar{\tau}_t, \bar{\tau}_t^*\}_{t=0}^{\infty}$$