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Debt Overhang and Sovereign Debt Restructuring*

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Abstract

Debt overhang is defined as a situation where a large amount of debt distorts the optimal investment decisions and discourages the government's efforts of the debtor country to undertake the necessary "adjustment policies".
In this paper I study some different strategies that can be used to solve the sovereign debt overhang problem. In particular, I consider two strategies based on a debt restructuring process, via haircut or rescheduling, and a third one based on conditional-additional lending. This strategy relies on the idea that the debtor country can get new lending from the existing creditors, in order to undertake investments that can affect the productivity shock distribution in a positive way (or reduce the probability of default).
The aim of this paper is to study the consequences, deriving from the three strategies described, on the incentives to invest in a "troubled country". According to these consequences and under some specific conditions, I rank the three strategies in order to see which is the most effective one. In particular, I find that if the change in investments due to the conditional-additional lending makes the probability of default low in this scenario, the conditional lending strategy will be the most effective one. Basically, this paper might help the policy-makers to implement the right intervention according to the specific scenarios considered.

JEL Codes: C78, F34, H63

Keywords: Debt Overhang, Debt Restructuring, Nash Bargaining, Haircut, Rescheduling, Conditional Lending.

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1 Introduction

In the past, debt accumulation and consequent default problems were considered an issue just for the developing and poor countries. Recently instead also some advanced economies have started to suffer from an extensive debt accumulation (Figure 1). The last decade indeed, due to the financial crisis of 2007 and the following sovereign debt crisis of 2009, has been characterized for a huge increase in debt in several countries.

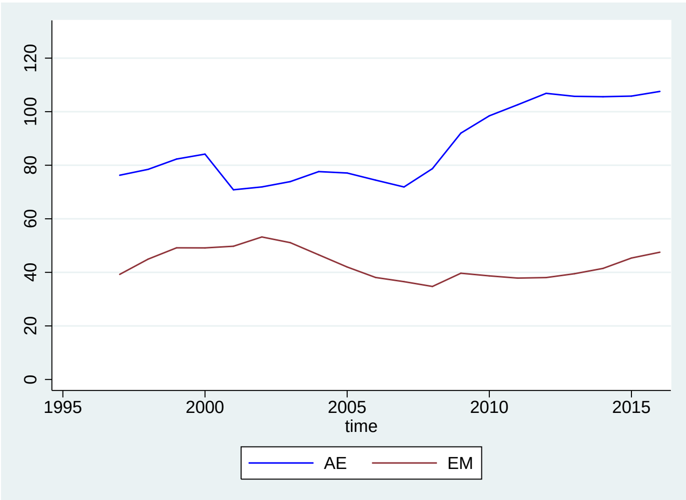


Figure 1: General Government Gross Debt as a percent of GDP: Advanced (35) and Emerging Market and Middle-Income Economies (42), 1997-2016. Source: IMF Fiscal Monitor.

These events have re-focused the attention on problems related to an excessive debt accumulation and on the possible policy interventions able to solve them.
In this regard, new attention has been posed on the debt overhang theory which analyzes the disincentive effect that an high level of debt produces on investments. Basically, a debt overhang scenario occurs when a debtor country, due to the "debt servicing obligations", can benefit only of a small portion of the profits deriving from the investments undertaken. In such a situation, debt is perceived as a tax on country's resources and it disincentives investments. For the same reason, a debt overhang condition is likely to discourage even the government's efforts of the debtor country to undertake the necessary "adjustment policies".

Considering these negative consequences deriving from an excessive debt accumulation, debt sustainability problems might arise. In this situation the use of fiscal policy in order to deal with cyclical crises might be undermined. This is the reason why new interest has been posed on sovereign debt restructuring (IMF (2013), IMF (2014), Buchheit et al. (2014)). The recent Greek case (with the largest restructuring episode in history) has re-animated the debate on how to restructure the public debt and on its possible consequences. Several proposal for a different form of debt restructuring have been advanced in the last years (Gianviti et al (2010), Paris and Wyplosz (2014)).

The aim of this paper is to build a two periods model with default risk that allows to analyze the effects of three different strategies, able to solve a debt overhang problem and to restore the incentives to invest in a highly indebted country, in order to find out the most effective one:

1. Sovereign debt restructuring: haircut
2. Sovereign debt restructuring: lenghtening of maturities
3. Conditional lending

With a large amount of outstanding debt, maintaining unchanged the nominal value of the debtor's obligations can produce negative consequences for both creditors and debtor. In such a situation and according to the debt overhang theory, a debt reduction could be an efficient strategy for both creditors and debtor countries. It should restore incentives to invest although it is not known which might be the most effective way to do that: haircut or lengthening maturities. A priori, it would seem reasonable to assume that an haircut can produce more effective results as it would deal with debt problems once and for all, solving definitely the problem of uncertainty. A lengthening of maturities process instead may just exacerbate the debt overhang condition by postponing the solution of the problem and without eliminating the uncertainty related to the borrower's financial conditions and repayment capacity.

The third strategy here analyzed is the so called "conditional lending" that is based on an additional financing, from existing lenders, for the debtor country. These creditors may, indeed, grant additional loans with the aim to protect the value of their claims toward the debtor country.

Just to give an example, after the eruption of the sovereign debt crisis and the provision of some additional lendings for Greece, debt restucturing interventions in the form of haircut became necessary in order to recover from a dramatic situation and to calm down the markets. This means that the strategy of providing new lending to the debtor country might work just under some specific conditions. I assume such new lending "conditional" because it is related to the debtor's commitment to make the necessary "adjustments" (investments). Moreover, I assume this new lending constrained by a specific credit ceiling and able to reduce the probability of default under specific conditions.

A comparison between these three strategies, with a focus on their consequences, might be useful in order to see which is the best way to deal with a debt overhang problem. In particular, given the current Greek crisis, such study might provide some useful information for the policy makers who are trying to solve the situation.

The paper proceeds as follows: Section 2 makes a literature review; Section 3 develops the theoretical model characterizing the restructuring through haircut, the restructuring through rescheduling and the conditional lending strategy; Section 4 concludes.

2 Related Literature

The debt overhang theory has been introduced for the first time by Myers (1977) in a corporate context. He defined debt overhang as a situation where the high level of debt distorts the possibilities for companies to make optimal future investment decisions. More specifically, a firm is less incentivated to invest because the benefits (i.e. future cash flows) deriving from the additional investment undertaken must be used largely to reimbourse the existing debt holders (rather than to pay the shareholders). The larger is the profit that goes to the creditors the lower is the incentive to invest and to suffer the cost of investment for the shareholders. Potential lenders might even decide to not finance the firm because of its high debt. Otherwise, they might ask for an higher interest rate in order to protect themselves. This reduces debtor's gain deriving from the investment undertaken thus creating a disincentive effect in the firm's investments policy. This reduced incentive to invest may then imply an underinvestment problem for firms with high levels of debt.

An example might be useful to clarify the debt overhang concept. Let's consider a firm with a debt of \$100, due the next year, and a future income of \$80 (i.e. the company will be in default the next year). Let's assume there is an investment opportunity that costs \$5 and produces \$15 next year [1] and the firm needs to raise funds to finance it. If the existing creditors are senior, then there will be no new investors willing to finance the project because the benefits will go just to the original creditors (who will increase their payoff to \$95) whereas the new ones will get nothing. If, instead, the project gives \$30 the next year, new investors might be interested in funding the project because they can get \$10 next year in return for the \$5 invested today. In conclusion, it is possible to finance investment just if the NPV of investing is larger than the debt overhang (here defined as the difference between assets and liabilities).

This concept of debt overhang used in the finance literature is similar to the concept used in the macro literature where the debt overhang has been analyzed mainly in the context of sovereign-debt crises. Indeed, due to the debt crises in the 80s and 90s, this theory has been extended in a sovereign context with the aim to explain the effects that the high debt had produced on the level of investment in the "less developed countries" (LDCs)[2]. In those

[1] assume the interest rate is zero so the project has a NPV of \$10.

[2] Several papers have analyzed these effects in that period (Krugman (1988), Krugman (1989), Borensztein (1990), Obstfeld and Rogoff (1996) and Sachs (1989)).

years, the highly indebted countries' low investments level and the low growth rate were often attributed, at least in part, to the high level of foreign debt. According to the debt overhang assumption, the accumulated debt discourages private sector's investment and the government's adjustment efforts because a possible improvement in the debtor country's performances determines mainly an increase in creditors' repayment.

From an empirical point of view, several studies tried to quantify the negative effects produced by an high level of debt. From on side, the literature focused on the effects on investments (Deshpande (1997)); from the other side, it focused on the direct effect on GDP growth rate. In this regard, some papers identified a non-linear relationship between debt and growth (Pattillo et al (2011); Clements et al (2003) and Reinhart et al (2012)) showing that high debt can affect negatively growth just if some thresholds are reached.

Several papers showed how desirable a debt relief was in the 80s and 90s for the LDCs. The debate was focused on the effects produced by the implemented restructuring programs. In particular, part of the literature showed the beneficial effects that some debt restructuring plans (the Brady Plan over all) produced in some countries. Arslanalp and Henry (2004) show the positive consequences that the Brady Plan had for both creditor and debtors countries. Specifically, they analyze these positive effects with respect to the stock prices for the debtor countries involved and the US commercial banks (i.e. the largest creditors at that time) with significant loan exposure towards those countries.
See Arslanalp and Henry (2006) for a literature review on the debt overhang problem.

In order to study the different possibilities available to restructure sovereign debt, it is useful to start from the survey of Das et al (2012a) and Das et al (2012b). In these papers, some interesting "stlylized facts" are described such as the frequency of restructuring, the number of London and Paris Club deals, the procedures used etc. regarding the several restructuring episodes that occurred between 1950 and 2010. In particular, some differences between the restructuring processes implemented by haircut and processes implemented by lengthening maturities are highlighted.

Even Reinhart and Trebesch (2015) analyze, but just from an empirical point of view, the different consequences produced by restructuring processes implemented via haircut or via lengthening maturities. In particular, they show that the debtor countries' conditions (economic growth or credit ratings) improve significantly just in case of debt write-offs. Different form of debt reliefs instead, such as lenghtening of maturities, are not generally followed by such results.

Let's see now how the three strategies, described in the introduction, have been treated so far.

In literature, a debt restructuring via haircut was initially analyzed by considering an exogenous debt reduction. Examples are given by Marchesi and Thomas (1999), Froot (1989) and Sachs (1989).

Literature has just recently started to consider an endogenous debt reduction, determined in a renegotiation process between creditors and debtor country. In this regard, Yue (2010) and Prokop (2012) are interesting papers because they consider a renegotiation between creditors and debtor in the form of a Nash Bargaining.

To my knowledge, no theoretical model has been built to study the effects, in a debt overhang context, of a debt restructuring via lengthening of maturities. However, the three periods model presented in Fernandez and Martin (2015) offers interesting food for thought for my goal especially in term of rollover risk. Bac (1999) instead considers an extreme lenghtening of maturity in the form of grace periods deriving as a Nash equilibrium creditors' strategy in a dynamic non cooperative game.

Finally, regarding the conditional lending (analyzed in particular by Obstfeld and Rogoff (1996), Sachs and Cohen (1982) and Krugman (1988)), most of the literature has argued that the strategy of granting new loans to an indebted country appears to be effective. Several studies have shown indeed that the liquidity constraints of debtor countries were just the main reason of low investment (Sachs and Cohen (1982), Froot (1989) and Borensztein (1988)). This may suggest that debt reduction operations should be accompanied by further loans in order to be more effective.

3 The Model

Let's consider a two periods model ($t = 1, 2$) for a debtor country that can be described by the following timeline:

At time $t = 1$ the country has an exogenous endowment Y_1 and an investment opportunity. The country then invests K_2 at $t = 1$ and gets a profit $AF(K_2)$ at time 2 (capital is assumed to depreciate at 100 %) with $F''(K_2) < 0$. "A" is the productivity shock and it is represented by a random variable that belongs to $[\underline{A}, \bar{A}]$, with $E(A) = 1$ and density probability function $\pi(A)$.
It is further assumed that the country has inherited debt D that expires in $t = 2$. It means that $AF(K_2)$ will be used in $t = 2$ for both consumption and debt repayment.
Let's assume that the country is risk neutral with the following utility function:

$$U_1 = C_1 + E(C_2) \tag{1}$$

To simplify, it is assumed that the discount factor is $\beta = 1$ and that the world interest rate

is $r = 0$.
Moreover, I assume that $\eta AF(K_2)$ represents the output reduction for the debtor country in case of default (as a deadweight loss) [3] with $0 < \eta < 1$.
At time $t = 1$, consumption is given by $C_1 = Y_1 - K_2$ whereas in $t = 2$ by $C_2 = AF(K_2) - P$. P represents the debtor's payment $P = min[D, \eta AF(K_2)]$ which depends on the realizations of A. From this expression it is possible to identify two intervals for the realizations of the variable A:

- if $A < \frac{D}{\eta F(K_2)}$ default will be more convenient for the debtor country as the output reduction in case of default is less than the cost of debt repayment;
- if $A > \frac{D}{\eta F(K_2)}$ the country repays its debt because it is more convenient compared to the loss of output that it would suffer in case of default.

To see the effect on the debt overhang problem of an haircut restructuring process, I assume that in case of default in $t = 2$, there is a re-negotiation between creditors and debtor in the form of Nash Bargaining. In this re-negotiation, an haircut on debt is determined in an endogenous way.
Let's see in more details how this re-negotiation process works.

3.1 Restructuring via haircut

In the Nash Bargaining there are two different scenarios that must be taken into account:

1. Agreement: the creditors receive as payment the new level of debt determined in the re-negotiation process whereas the debtor gets the difference between the output produced in the second period and what it must repay:
   $U^C = D(1-h)$
   $U^D = AF(K_2) - D(1-h)$

2. No Agreement: to simplify I assume that the creditors gets nothing whereas the debtor suffers a reduction in its output because of the default:
   $\bar{U}^C = 0$
   $\bar{U}^D = (1-\eta)AF(K_2)$

[3] $\eta AF(K_2)$ in literature can be considered both as a fraction of output of the debtor country that can be expropriated from creditors or just as an output reduction due to the default. In this paper I will consider it as an output reduction. Several papers study the consequent output reduction for a country in default deriving from trade sanctions (Rose (2005)), exclusion from financial markets (Cruces and C. (2013)), reduced credit to the private sector (Arteta and G. (2008))etc. The so called "issue linkages". For a review see Sandleris (2012)

where U^C and U^D represent respectively creditors' and debtor's utility functions in case of agreement ($\bar{U}^C$ and $\bar{U}^D$ in case of no agreement).
Thus, it is possible to build the creditors' $(U^C - \bar{U}^C)$ and debtor's surplus functions $(U^D - \bar{U}^D)$ that represent the utility gains of creditors and debtor country in case of agreement in the Nash Bargaining. The resulting Nash product Ω must be maximized with respect to h (i.e. the amount of haircut) considering θ as creditors' bargaining power and $(1-\theta)$ as debtor's bargaining power. This is a maximization problem with creditors' and debtor's surplus functions as constraints.

$$\begin{aligned} \max_{K_2} \quad & \Omega \\ \text{s. t.} \quad & U^C - \bar{U}^C \geq 0 \\ \text{s. t.} \quad & U^D - \bar{U}^D \geq 0 \end{aligned}$$

The result of this maximization is:

$$h = 1 - \theta\eta \frac{AF(K_2)}{D} \tag{2}$$

Computations are showed in Appendix A.
Thus, the amount of debt reduced that must be paid at the end of the Nash Bargaining process is:

$$D(1-h) = \theta\eta AF(K_2)$$

If $\theta = 0$, there will be a 100% haircut: the debtor pays 0.
If $\theta = 1$, there will be a 0% haircut: the debtor pays $\eta AF(K_2)$.
If $0 < \theta < 1$, then $0 < h < 1$ and the debtor pays an amount $0 < D(1-h) < \eta AF(K_2)$.
In conclusion, as we can see from equation (2), it is possible to claim that:

- an increase in creditors' bargaining power reduces the level of h;
- an higher output reduction in case of default negatively influences the level of h (as a sort of disincentive to default);
- an higher D implies the necessity of an higher haircut.

Thanks to the result of the restructuring process just found, consumption in the second period can be re-written as:

$$C_2 = \begin{cases} C_2^R = AF(K_2) - D(1-h) \\ C_2^N = AF(K_2) - D \end{cases}$$

With C_2^R consumption in case of restructuring and C_2^N consumption in case of no default. Maximizing the (1) with respect to K_2 and taking into account the result deriving from the Nash bargaining produces the following result:

$$F'(K_2)\left[1 - \theta\eta\int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)\right] = 1 \tag{3}$$

Computations are showed in Appendix A1.
$\theta\eta\int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)dA$ represents how the marginal change in K_2 affects the payment in case of haircut (if there is a change in the amount to be paid) and $\pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)$ represents the difference between the repayment change in case of no haircut and in case of haircut due to a change in K_2 (that affects the default probability).

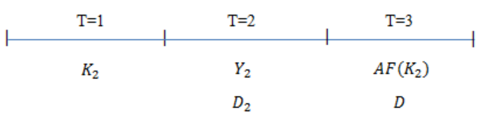
Thus, it is possible to claim that the final result, in terms of incentives to invest, depends on the value of the parameter θ: the lower the value of this parameter, the higher will be the incentives to invest. This is because the smaller the θ, the greater will be the value assumed by h. In other words, the effectiveness of the haircut result depends on the creditors' bargaining power.
In order to see how the creditors' bargaining power (θ) affects the relation between debt and investment it is useful to compute the implicit differentiation of (3).

$$\frac{dK_2}{dD} = \frac{\left[\frac{\theta D}{\eta F(K_2)^2}F'(K_2)\pi\left(\frac{D}{\eta F(K_2)}\right) + F'(K_2)(1-\theta)\frac{D}{\eta F(K_2)^2}\left(\pi'\left(\frac{D}{\eta F(K_2)}\right)\frac{D}{\eta F(K_2)} + 2\pi\left(\frac{D}{\eta F(K_2)}\right)\right)\right]}{U''(K_2)}$$

that is negative if $U''(K_2) < 0$.
Computations are showed in Appendix A2.

3.2 Restructuring via rescheduling

A second strategy here analyzed is related to a restructuring process via lengthening of maturities. In order to model a rescheduling process, I add an intermediate period where the debtor country has an exogenous endowment Y_2 and it is due a debt payment D_2. The following timeline describes the current situation:



It is assumed that the debtor country doesn't have the necessary resources to repay this amount of debt ($Y_2 < D_2$). Therefore, its only available strategy is to default on $t = 2$ and it would imply an output loss for the debtor country both in $t = 2$ and $t = 3$.

From creditors' point of view, it is surely preferable to avoid a default as it would involve a total loss of their claims. Thus, creditors give the opportunity to restructure the debt through a lengthening of maturities. It means to postpone the payment of D_2 from $t = 2$ to $t = 3$ through a debt rollover. Then, in the third period, since the debtor country gets profits from investment and there is the realization of the productivity shock, creditors might be fully repaid.

Since I am introducing the debt rollover's interest rate, it is necessary to consider a risk free interest rate $r_F \neq 0$ and a risky interest rate as a convex function of the level of debt $r = r(D)$. Thus, I assume that the debt is rolled over at the rate $1 + r$. The interest rate $1 + r$, defined in such a way, reflects the riskiness of the debtor. If the debtor country has a low level of debt, then the rollover implies a lower interest rate. Vice versa debt is rolled over at an higher interest rate (because of the higher risk).

Thus, in $t = 3$ the debtor will pay the $min[D^*, \eta AF(K_2)]$ with $D^* = D + D_2(1 + r)$ that represents the total debt that must be paid.

If $A < \frac{D^*}{\eta F(K_2)} = A^*$ default will be more convenient for the debtor country since the output reduction in case of default would be less than the cost of paying off the debt.

The other way round, if $A > \frac{D^*}{\eta F(K_2)}$ the country will pay back its debt because it will be more convenient compared to the output loss that it would suffer in case of default.

Since now we are considering an interest rate different from zero, it is necessary to introduce a discount factor $\beta = 1/(1 + r(D))$ that is different from 1.

The utility function is:

$$U = Y_1 - K_2 + \beta E(Y_2) + \beta^2 \left[\int_{\underline{A}}^{A^*} AF(K_2)(1-\eta)\pi(A)dA + \int_{A^*}^{\bar{A}} (AF(K_2) - D^*)\,\pi(A)dA \right]$$

I maximize this utility function with respect to K_2 and the final result is:

$$\beta^2 F'(K_2) \left[1 - \eta \int_{\underline{A}}^{\frac{D + D_2(1+r)}{\eta F(K_2)}} A\pi(A)dA \right] = 1$$

Computations are showed in Appendix B.

So far, I considered a general case of debt rollover without assuming any form of debt relief related to the lengthening of maturities. Then, I can assume that in order to help the debtor country, the debt rollover is made at the risk-free interest rate $(1 + r_F)$ as a sort of "concessional interest rate" and not at $(1 + r)$.

Thus:

$$\beta^2 F'(K_2) \left[1 - \eta \int_{\underline{A}}^{\frac{D + D_2(1+r_F)}{\eta F(K_2)}} A\pi(A)dA \right] = 1 \qquad (4)$$

and according to this equation K_2 will be greater than before.

Now, since I want to compare this result with that one obtained in the case of haircut (3) (assuming three periods), I have that:

$$\left[1-\theta\eta\int_{\underline{A}}^{\frac{D+D_2(1+r)}{\eta F(K_2)}} A\pi(A)dA-\pi\left(\frac{D+D_2(1+r)}{\eta F(K_2)}\right)\frac{(D+D_2(1+r))^2}{\eta F(K_2)^2}(1-\theta)\right]$$

$$\gtrless\left[1-\eta\int_{\underline{A}}^{\frac{D+D_2(1+r_F)}{\eta F(K_2)}} A\pi(A)dA\right]$$

on the left side we have the result in case of haircut (assuming three periods and debt rollover), on the right that one for rescheduling. The higher one will imply higher incentive to invest.

We can assume an equivalence for a given $\underline{\theta}$ and after simplifying we get:

$$-\underline{\theta}\int_{\underline{A}}^{A2} A\pi(A)dA-\pi(A2)A2^2(1-\underline{\theta})=-\int_{\underline{A}}^{A1} A\pi(A)dA$$

with $A2=\frac{D+D_2(1+r)}{\eta F(K_2)}$ and $A1=\frac{D+D_2(1+r_F)}{\eta F(K_2)}$ and $A2>A1$. Rearranging:

$$\underline{\theta}\left[-\int_{\underline{A}}^{A2} A\pi(A)dA+\pi(A2)A2^2\right]=\left[-\int_{\underline{A}}^{A1} A\pi(A)dA+\pi(A2)A2^2\right]$$

Since: $\int_{\underline{A}}^{A^*} A\pi(A)dA>\pi(A^*)A^{*2}$ as I proved in Appendix A1, then $\int_{\underline{A}}^{A2} A\pi(A)dA>\pi(A2)A2^2$ and the parts in the parenthesis are negative (because θ must be a positive value). Moreover, we know that $\int_{\underline{A}}^{A2} A\pi(A)dA>\int_{\underline{A}}^{A1} A\pi(A)dA$.

Thus, if $\theta<\underline{\theta}$:

$$\theta\left[-\int_{\underline{A}}^{A2} A\pi(A)dA+\pi(A2)A2^2\right]>\left[-\int_{\underline{A}}^{A1} A\pi(A)dA+\pi(A2)A2^2\right]$$

and haircut is better than lengthening of maturities in terms of incentive to invest.

$\underline{\theta}$ can be then considered as a threshold for creditor's bargaining power:

- if $\theta<\underline{\theta}$ an haircut will be preferred.
- if $\theta>\underline{\theta}$ a lengthening process will be preferred.

3.3 Conditional-Additional Lending

An high debt can adversely affect investments in two ways: through the debt overhang condition above examined and through the credit rationing.

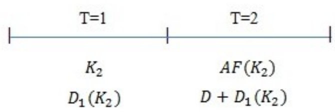
The credit rationing derives from the presence of high interest rates that an highly indebted country faces into the international financial markets, because of its weak standing.

It should be verified if the main reason for the low level of investment are the liquidity

constraints or the debt overhang condition.
However it is possible to assume that, despite the difficult situation of the debtor country, the existing creditors might be still willing to grant additional loans. In a defensive lending perspective (Krugman (1989)) indeed, existing creditors may be willing to further finance the debtor with the aim to protect the value of their claims. In others words, they hope that in the future the debtor will become able of repay its debt.

There is also another reason why the existing creditors might be interested in providing new lending to the debtor country. Indeed, they might believe that if the new additional lending is used to undertake some productive investments (i.e. conditional lending) it could reduce the debtor country's probability of default. If such consequence is proved to be true, then it might be interesting from a creditor's point of view to compare two strategies: haircut vs conditional-additional lending. In other words: is it better to allow an haircut for a given probability of default or to lend new money (with an increase in credit exposure) since this reduces the probability of default?

Thus, I assume a precommitment to invest for the debtor country (Obstfeld and Rogoff (1996)): firstly the debtor establishes the amount K_2 to invest and secondly the creditors, being able to observe the value of K_2, decide the amount of new lending. Therefore, it is possible to consider the new loans as a function of the level of investments undertaken by the debtor country: $D_1(K_2)$.
The new total debt that must be repaid at end of period 2 will be then $D^* = D_1(K_2) + D$. The following timeline describes the current situation:



Following this reasoning, creditors would be willing to lend more and more with the increase in the amount invested by the debtor country. Actually it is reasonable to assume that there is a maximum (credit ceiling) on the total amount of new loans that the creditors will be willing to grant. Specifically, creditors will provide new loans with the aim to not alter debtor's repayment incentives. This corresponds to a situation where the nominal value of debt that the debtor will have to repay will be equal or lower than the cost of default (calculated according to the expectations of creditors). Then, the credit ceiling considered will be given by:

$$D_1(K_2) + D \leq \eta E[A] F(K_2)$$

According to the last expression, existing creditors will be willing to provide new loans up to the point where the total debt accumulated by the debtor country, will be exactly equal to the expected output reduction that the debtor country would suffer in case of default (this is the maximum amount of new loans).

Let's see now the effect of the additional lending and the consequent additional investment over the default probability. The probability of default in case of conditional lending is given by:

$$\int_{\underline{A}}^{\frac{D+D_1(K_2)}{\eta F(K_2)}} \pi(A)dA$$

I compute the derivative of the default probability with respect to K_2 and I find that it is negative if and only if:

$$\frac{\frac{\delta D_1(K_2)}{\delta K_2}}{\eta} < F'(K_2) \tag{5}$$

Computations are showed in Appendix C.

If the (5) holds, it implies that the additional investment deriving from the additional lending will reduce the default probability notwithstanding the increase in debt.

This condition is a sort of restriction for the provision of new capital from the existing creditors. It means that the creditors provide new lending for each additional unit of investment undertaken by the debtor country but in a lesser extent with respect the productivity that this additional unit of investment produces.

Let's consider the debtor country's utility function in the case of conditional lending.
I assume that the upper bound of the shock probability distribution is function of the investment undertaken: $\bar{A}(K_2)$. In this way, an increase in investments will produce an expansion to the right of the productivity shock distribution (because of the increase in its upper bound).

$$U = Y_1 - K_2 + D_1(K_2) + \int_{\underline{A}}^{\frac{D^*}{\eta F(K_2)}} \left[AF(K_2)(1-\eta)\right]\pi(A)dA +$$
$$+ \int_{\frac{D^*}{\eta F(K_2)}}^{\bar{A}(K_2)} \left[AF(K_2) - (D_1(K_2)+D)\right]\pi(A)dA$$

I maximize this utility function with respect to K_2 assuming the credit ceiling as binding and I get:

$$F'(K_2)\left[E(A) - \eta\int_{\underline{A}}^{A^*} A\pi(A)dA + \eta E(A)\int_{\underline{A}}^{A^*}\pi(A)dA\right] =$$
$$1 - F(K_2)\pi(\bar{A}(K_2))\bar{A}'(K_2)\left[\bar{A}(K_2) - \eta E(A)\right]$$

I need now some assumptions for the productivity shock probability distribution.

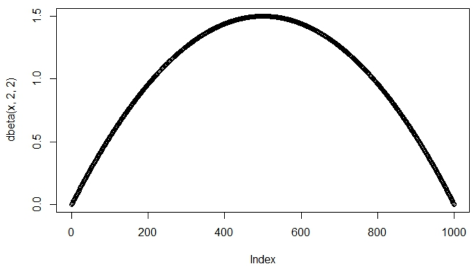
To do that, I consider that the probability of default in case of conditional lending can be written as:

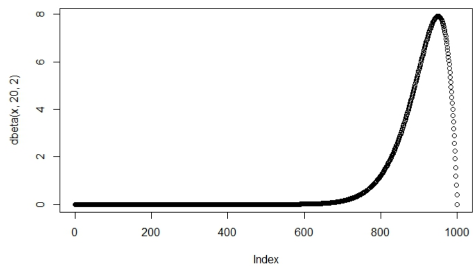
$$\int_{\underline{A}}^{\frac{D+D_1(K_2)}{\eta F(K_2)}} \pi(A)dA$$

or

$$1-\int_{\frac{D+D_1(K_2)}{\eta F(K_2)}}^{\bar{A}(K_2)} \pi(A)dA$$

and the derivative of these two expressions with respect to K_2 must be the same. Thus, as showed in Appendix C1, it must be that: $\pi(\bar{A}(K_2)) = 0$ and/or $\bar{A}'(K_2) = 0$.
According to these results, it might be useful to consider a Beta distribution $B(\alpha, \beta)$ between $\underline{A}$ and $\bar{A}$ with the parameter α considered as a concave function of investments ($\alpha = f(K_2)$) and the parameter β as a convex function of debt ($\beta = f(D)$).
It means that the probability to get positive values of the productivity shock is bigger with the increase in the investments undertaken.





The graph on the left shows the case of a Beta with $\alpha = \beta$ and expected value of 1. The second graph shows the consequences of an increase in parameter α (i.e. increase in K_2). An increase in parameter β would instead produce an opposite result (movement to the left).
According to this distribution for the productivity shock, it makes sense to increase investments, thanks to the conditional lending, because it would imply an increase in the probability to get higher values for A.

If we consider the Beta distribution, then the result from the utility function maximization with respect to K_2 becomes:

$$F'(K_2)\left[E[A] - \eta\int_{\underline{A}}^{A^*} A\pi(A)dA + \eta E[A]\int_{\underline{A}}^{A^*} \pi(A)dA\right] = 1 \tag{6}$$

Computations are showed in Appendix C1.
$\eta E[A]\int_{\underline{A}}^{A^*} \pi(A)dA$ represents the extra benefits deriving from the possibility to get the additional lending of the existing creditors.
If the change in K_2 (due to the additional lending) brings the default threshold A^* below 1 (i.e. it makes the probability of default low), then the extra benefit is surely bigger than the cost of default and investments will be higher. Vice versa if the cost of default has a

larger effect, investments will decrease. In other words: if the investment is effective (i.e. it makes the probability of default low) then the conditional lending strategy will an effective strategy.

3.4 Creditors' Point of View

What is the best strategy from creditors' point of view when a risk of default for the debtor country is appearing?
1) haircut taking $D(1-h)$ with a given probability of default or
2) conditional lending (with an increase in exposure) that gives the possibility to be fully repaid because of the higher investments ($K^* > K_2$).
What are the main determinants that can affect the choice between haircut and conditional lending?

- creditors' impatience and their risk aversion;
- default probability: creditors would prefer an haircut if this probability is high (the same for high level of debt). Otherwise, in case of high probability of default, the debtor would need a huge effort/investment (K_2) to bring this probability back to normal level but it might not have the economic or political capability to do that;
- the possibility to reduce the default probability through conditional lending;
- the debtor country would need an investment/adjustment plan considered efficient from creditors;
- the timing of intervention: if there is the possibility to intervene when the default probability is still low, it could be useful a conditional lending strategy. On the other hand, if it is too late and the default probability is already high it might be necessary an haircut.

3.5 Ranking Strategies

Let's make a comparison between all the three strategies described (assuming the same conditions) in order to see which is the best solution able to restore incentives to invest.
The optimal investment functions are:
<u>Haircut</u>

$$\beta^2 F'(K_2)\left[1-\theta\eta\int_{\underline{A}}^{\frac{D+D_2(1+r)}{\eta F(K_2)}} A\pi(A)dA - \pi\left(\frac{D+D_2(1+r)}{\eta F(K_2)}\right)\frac{(D+D_2(1+r))^2}{\eta F(K_2)^2}(1-\theta)\right]=1$$

<u>Rescheduling</u>

$$\beta^2 F'(K_2)\left[1-\eta\int_{\underline{A}}^{\frac{D+D_2(1+r_F)}{\eta F(K_2)}} A\pi(A)dA\right]=1$$

<u>Conditional Lending</u>

$$\beta^2 F'(K_2)\left[E(A)-\eta\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} A\pi(A)dA+\eta E(A)\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} \pi(A)dA\right]=1$$

Let's study two different cases:

1. If $\eta E(A)\int_{\underline{A}}^{\frac{D+D_1(K_2)}{\eta F(K_2)}} \pi(A)dA > \eta\int_{\underline{A}}^{\frac{D+D_1(K_2)}{\eta F(K_2)}} A\pi(A)dA$ (i.e. if the change in K_2 makes the default threshold $A^* < 1$), then the conditional lending will increase investments more than haircut (with $E(A) > 1$):

$$\left[E(A)-\eta\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} A\pi(A)dA+\eta E(A)\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} \pi(A)dA\right]>$$

$$\left[1-\theta\eta\int_{\underline{A}}^{\frac{D+D_2(1+r)}{\eta F(K_2)}} A\pi(A)dA-\pi\left(\frac{D+D_2(1+r)}{\eta F(K_2)}\right)\frac{(D+D_2(1+r)^2}{\eta F(K_2)^2}(1-\theta)\right]$$

   and more than rescheduling:

$$\left[E(A)-\eta\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} A\pi(A)dA+\eta E(A)\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} \pi(A)dA\right]>$$

$$\left[1-\eta\int_{\underline{A}}^{\frac{D+D_2(1+r_F)}{\eta F(K_2)}} A\pi(A)dA\right]$$

   Thus, conditional lending is the best strategy able to incentive investments.
   Then in order to compare haircut and rescheduling I use the threshold for $\underline{\theta}$ (as showed in Paragraph 3.2):

   - if $\theta < \underline{\theta}$ haircut is more effective.
   - if $\theta < \underline{\theta}$ rescheduling is more effective.

2. Let's consider conditional lending and rescheduling (with $E(A) > 1$):

$$\left[E(A)-\eta\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} A\pi(A)dA+\eta E(A)\int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} \pi(A)dA\right]\gtrless$$

$$\left[1-\eta\int_{\underline{A}}^{\frac{D+D_2(1+r_F)}{\eta F(K_2)}} A\pi(A)dA\right]$$

I compare the default threshold in case of conditional lending and rescheduling:

$$\frac{D + D_1(K_2)(1+r)^2}{\eta F(K_2)} \gtrless \frac{D + D_2(1+r_F)}{\eta F(K_2)}$$

rearranging:

$$D_1(K_2)(1+r)^2 \gtrless D_2(1+r_F)$$

I assume there is an equivalence for a given $\bar{D}_2$.
If $D_2 = \bar{D}_2$ then the probability of default is the same in both cases.
If $D_2 > \bar{D}_2$ then the probability of default is bigger in case of rescheduling.
If $D_2 < \bar{D}_2$ then the probability of default is bigger in case of conditional lending.

- Let's assume $\underline{D_2 = \bar{D}_2}$.
  The probabilities of default are the same and I get:

$$E(A) + \eta E(A) \int_{\underline{A}}^{\frac{D+D_1(K_2)(1+r)^2}{\eta F(K_2)}} \pi(A)dA > 1$$

  thus conditional lending is the best solution.

- Let's assume $\underline{D_2 > \bar{D}_2}$.
  The probability of default is bigger in case of rescheduling.
  If the change in K_2 makes the default threshold $A^* < 1$ then conditional lending is the best solution. But even if the change in K_2 makes the default threshold $A^* > 1$ then conditional lending is the best solution (because the debt burden in case of rescheduling is larger).

- Let's assume $\underline{D_2 < \bar{D}_2}$.
  The probabilitiy of default is bigger in case of conditional lending.
  If the change in K_2 makes the default threshold $A^* < 1$ then conditional lending is the best solution. Conversely, if the change in K_2 makes the default threshold $A^* > 1$ then the result is uncertain (it depends on parameter's calibration).

4 Conclusion

This paper shows the effectiveness of three different strategies that can be used to solve the debt overhang problem and to restore the incentives to invest in a troubled country.
Two strategies are based on debt relief interventions. The first one is based on an haircut (with the amount of debt that is reduced endogenously through a Nash Bargaining process)

whereas the second one consists in debt rescheduling (with the rollover interest rate that is assumed risk free).
The third strategy is based on conditional-additional lending. It means that the existing creditors provide new lending for the debtor country (the so called defensive lending).
One important result of this work is to show the optimal investment function for each strategy analysed.
Then I make some comparisons between these strategies.
The first result is that a restructuring process through haircut allows to restore the incentives to invest in a stronger way compared to a restructuring through maturities extension. This result depends on one important variable: the creditors' bargaining power in the re-negotiation process with the debtor country. More specifically, if creditors' bargaining power is lower than a specific threshold (i.e. creditors are weak) then haircut will be better off with respect to lengthening of maturities in terms of restoring incentives to invest.
Then, I compare haircut and rescheduling with conditional lending and I find that if the change in investment (due to the additional lending) makes the probability of default low then conditional lending will imply higher investments with respect to both haircut and rescheduling.
In conclusion, I am able to build a ranking for the described strategies that would be helpful for the policy makers struggling to solve a debt crisis. In particular, if the extra benefits deriving from additional lending are greater than the cost of payment, then the conditional lending strategy is the best intervention in order to restore incentive to invest. Then, if the creditors' bargaining power is below the threshold identified in the paper, haircut will be the second best solution and rescheduling the last one.
The other way round, if the extra benefits deriving from additional lending are lower than the cost of payment, conditional lending will be always the preferred solution except in the case where the probability of default in case of conditional lending is bigger than in the other cases. In such a situation the final result is uncertain (it depends on parameter's calibration).

Then, another interesting result of this paper is that conditional-additional lending from existing creditors can reduce the probability of default under specific circumstances (notwithstanding the increase in debt). Thus, it might be interesting from creditors' point of view to ask themselves: is it better to give an haircut taking back a certain reduced amount of debt or to increase lending since it gives the possibility to be fully repaid?

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Appendix A

This Appendix shows how to compute the haircut deriving from the Nash Bargaining.
The creditors' $(U^C - \bar{U}^C)$ and debtor's surplus functions $(U^D - \bar{U}^D)$ represent the utility gains of creditors and debtor country in case of agreement in the Nash Bargaining.
The creditors' surplus deriving from the re-negotiation is then:

$$\left(U^C - \bar{U}^C\right) = D(1-h)$$

whereas debtor's surplus is:

$$\left(U^D - \bar{U}^D\right) = AF(K_2) - D(1-h) - (1-\eta)AF(K_2)$$

The Nash product is:

$$\Omega = \left(U^C - \bar{U}^C\right)^{\theta} \left(U^D - \bar{U}^D\right)^{1-\theta}$$

that must be maximized with respect to h considering θ as creditors' bargaining power and $(1-\theta)$ as debtor's bargaining power. This is then a constrained maximization problem:

$$\begin{aligned} \max \quad & \Omega \\ \text{s. t.} \quad & U^C - \bar{U}^C \geq 0 \\ \text{s. t.} \quad & U^D - \bar{U}^D \geq 0 \end{aligned}$$

The Lagrangian is:

$$L = [D(1-h)]^{\theta} [AF(K_2) - D(1-h) - (1-\eta)AF(K_2)]^{1-\theta} - \lambda_1(D(1-h)) - \lambda_2(-D(1-h) + \eta AF(K_2))$$

I compute the derivative of the Lagrangian with respect to h and I get:

$$\frac{\delta L}{\delta h} = \theta(-D)[U^C - \bar{U}^C]^{\theta-1}[U^D - \bar{U}^D]^{1-\theta} + [U^C - \bar{U}^C]^{\theta}(1-\theta)D[U^D - \bar{U}^D]^{-\theta} + \lambda_1 D - \lambda_2 D = 0$$

with the two constraints:

$$D(1-h) \geq 0 \quad \wedge \quad \lambda_1(D(1-h)) = 0$$

$$D(1-h) \leq \eta AF(K_2) \quad \wedge \quad \lambda_2[(-D(1-h) + \eta AF(K_2)] = 0$$

$\lambda_1 > 0$ is not an acceptable solution because it would imply $D(1-h) = 0$ that is a 100% haicut. $\lambda_2 > 0$ is not an acceptable solution because it would imply $D(1-h) = \eta AF(K_2)$ that is a situation in which the debtor would be indifferent between payback and default. Therefore, the only acceptable solutions are $\lambda_1 = 0$ e $\lambda_2 = 0$.

To simplify, I multiply both sides of the equation for $\left(\dfrac{U^D - \bar{U}^D}{U^C - \bar{U}^C}\right)^{\theta}$ and I after some computations I get:

$$h = 1 - \theta\eta \frac{AF(K_2)}{D}$$

Appendix A1

This Appendix shows how to get the result in the haircut case.

Taking into account the result of the Nash Bargaining, the utility function is:

$$U = Y_1 - K_2 + \int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} AF(K_2)(1-\theta\eta)\pi(A)dA + \int_{\frac{D}{\eta F(K_2)}}^{\bar{A}} \left(AF(K_2) - D\right)\pi(A)dA$$

$$U = Y_1 - K_2 + F(K_2)(1-\theta\eta)\int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)dA + F(K_2)\int_{\frac{D}{\eta F(K_2)}}^{\bar{A}} A\pi(A)dA - D\int_{\frac{D}{\eta F(K_2)}}^{\bar{A}} \pi(A)dA$$

I derive with respect to K_2:

$$\frac{\delta U}{\delta K_2} = -1 + F'(K_2)(1-\theta\eta)\int_{\underline{A}}^{A^*} A\pi(A)dA + F(K_2)(1-\theta\eta)\frac{D}{\eta F(K_2)}\pi\left(\frac{D}{\eta F(K_2)}\right)\left(-\frac{D\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) +$$

$$+F'(K_2)\int_{A^*}^{\bar{A}} A\pi(A)dA + F(K_2)\left(-\frac{D}{\eta F(K_2)}\right)\pi\left(\frac{D}{\eta F(K_2)}\right)\left(-\frac{D\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) +$$

$$-D\left(-\pi\left(\frac{D}{\eta F(K_2)}\right)\right)\left(-\frac{D\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) = 0$$

and after some computations I get:

$$F'(K_2)\left[1 - \theta\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)\right] = 1 \qquad (7)$$

Now I want to compare this result with OR(1996)'s original result:

I want to show that the haircut's result implies higher incentives to invest.

$$\left[1 - \theta\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)\right] \gtrless \left[1 - \eta\int_{\underline{A}}^{A^*} A\pi(A)dA\right]$$

The higher one will imply higher incentive to invest (in the right parenthesis there is the OR's result).

Let's suppose the equivalence:

$$\eta\theta\int_{\underline{A}}^{A^*} A\pi(A)dA + \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta) = \eta\int_{\underline{A}}^{A^*} A\pi(A)dA$$

If I consider this equivalence it means that I am indifferent between the situation with haircut and the situation without haircut: the payment would be the same in both cases. To have that haircut is beneficial with respect to the normal situation, I must impose that:

$$+\eta\theta\int_{\underline{A}}^{A^*} A\pi(A)dA + \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta) < \eta\int_{\underline{A}}^{A^*} A\pi(A)dA$$

$$\theta\left(\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}\right) < \left(\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}\right)$$

This is always verified $\quad \forall\theta < 1$ if $\quad \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2} < \eta\int_{\underline{A}}^{A^*} A\pi(A)dA$

Now I compute the second derivative to prove that equation (9) represents a maximum.

$$F''(K_2) - F''(K_2)\theta\eta\int_{\underline{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A)dA - F'(K_2)\theta\eta\frac{D}{\eta F(K_2)}\pi\left(\frac{D}{\eta F(K_2)}\right)\left(-\frac{D\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) +$$

$$-F''(K_2)\pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta) - F'(K_2)(1-\theta)\left[\pi'\left(\frac{D}{\eta F(K_2)}\right)\left(-\frac{D\eta F'(K_2)}{\eta^2 F(K_2)^2}\right)\frac{D^2}{\eta F(K_2)^2} + \right.$$

$$\left. +\pi\left(\frac{D}{\eta F(K_2)}\right)\left(-\frac{2D^2\eta F(K_2)}{\eta^2 F(K_2)^4}\right)\right] = 0$$

$$F''(K_2)\left[1 - \theta\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)\right] +$$

$$+F'(K_2)\frac{D^2}{\eta F(K_2)^3}\left[\frac{\theta\eta}{\eta}F'(K_2)\pi\left(\frac{D}{\eta F(K_2)}\right) + (1-\theta)\pi'\left(\frac{D}{\eta F(K_2)}\right)\frac{DF'(K_2)}{\eta F(K_2)} + 2(1-\theta)\pi\left(\frac{D}{\eta F(K_2)}\right)\right] = 0$$

$$F''(K_2)\left[1 - \theta\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)\right] +$$

$$+F'(K_2)\frac{D^2}{\eta F(K_2)^3}\left[\theta F'(K_2)\pi\left(\frac{D}{\eta F(K_2)}\right) + (1-\theta)\pi'\left(\frac{D}{\eta F(K_2)}\right)\frac{DF'(K_2)}{\eta F(K_2)} + 2(1-\theta)\pi\left(\frac{D}{\eta F(K_2)}\right)\right] < 0$$

This condition needs not hold for all K_2 but it must hold at the optimal investment level. It is negative since: $F''(K_2) < 0$ and $\left[1 - \theta\eta\int_{\underline{A}}^{A^*} A\pi(A)dA - \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{D^2}{\eta F(K_2)^2}(1-\theta)\right] > 0$ and also the second term is positive.

Appendix A2
This Appendix shows how to compute the implicit differentiation.
Let's consider (3) as F.

$$\frac{dK_2}{dD} = -\frac{\delta F/\delta D}{\delta F/\delta K_2} =$$

$$= \frac{-\left[-\theta\eta\frac{D}{\eta F(K_2)}\pi\left(\frac{D}{\eta F(K_2)}\right)F'(K_2)\frac{1}{\eta F(K_2)} - F'(K_2)(1-\theta)\left(\pi'\left(\frac{D}{\eta F(K_2)}\right)\frac{1}{\eta F(K_2)}\frac{D^2}{\eta F(K_2)^2} + \pi\left(\frac{D}{\eta F(K_2)}\right)\frac{2D}{\eta F(K_2)^2}\right)\right]}{U''(K_2)}$$

$$= \frac{\left[\frac{\theta D}{\eta F(K_2)^2}F'(K_2)\pi\left(\frac{D}{\eta F(K_2)}\right) + F'(K_2)(1-\theta)\frac{D}{\eta F(K_2)^2}\left(\pi'\left(\frac{D}{\eta F(K_2)}\right)\frac{D}{\eta F(K_2)} + 2\pi\left(\frac{D}{\eta F(K_2)}\right)\right)\right]}{U''(K_2)}$$

Appendix B
This appendix shows how to get the result in the lengthening of maturities case.
The utility function is:

$$U = Y_1 - K_2 + \beta E(Y_2) + \beta^2\left[\int_{\underline{A}}^{A^*} AF(K_2)(1-\eta)\pi(A)dA + \int_{A^*}^{\bar{A}} \left(AF(K_2) - D^*\right)\pi(A)dA\right]$$

I maximize this utility function with respect to K_2:

$$\frac{\delta U}{\delta K_2} = -1 + \beta^2 F'(K_2)(1-\eta)\int_{\underline{A}}^{A^*} A\pi(A)dA + \beta^2(1-\eta)F(K_2)\frac{D^*}{\eta F(K_2)}\pi\left(\frac{D^*}{\eta F(K_2)}\right)\left(-\frac{D^*\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) +$$
$$+\beta^2 F'(K_2)\int_{A^*}^{\bar{A}} A\pi(A)dA + \beta^2 F(K_2)\left(-\frac{D^*}{\eta F(K_2)}\right)\pi\left(\frac{D^*}{\eta F(K_2)}\right)\left(-\frac{D^*\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) +$$
$$+\beta^2 D^*\pi\left(\frac{D^*}{\eta F(K_2)}\right)\left(-\frac{D^*\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) = 0$$

and after simplifying:

$$\beta^2 F'(K_2)\left[1 - \eta\int_{\underline{A}}^{\frac{D+D_2(1+r)}{\eta F(K_2)}} A\pi(A)dA\right] = 1$$

Appendix C
This Appendix shows that the probability of default is decreasing for an increase in K_2.

The probability of default in case of conditional lending is given by:

$$\int_{\underline{A}}^{\frac{D+D_1(K_2)}{\eta F(K_2)}} \pi(A)dA$$

The derivative with respect to K_2 is:

$$\frac{\delta}{\delta K_2} = \pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)\left(\frac{\delta\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)}{\delta K_2}\right)$$

That is negative if and only if :

$$\left(\frac{\delta\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)}{\delta K_2}\right) < 0$$

It implies that:

$$\left(\frac{\delta\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)}{\delta K_2}\right) = \frac{\frac{\delta D_1(K_2)}{\delta K_2}\eta F(K_2) - (D+D_1(K_2))\eta F'(K_2)}{\eta^2 F(K_2)^2} < 0$$

It must be that:

$$\frac{\delta D_1(K_2)}{\delta K_2}F(K_2) < (D+D_1(K_2))F'(K_2)$$

Assuming that the credit ceiling is binding (and $E(A)=1$), I get:

$$\frac{\frac{\delta D_1(K_2)}{\delta K_2}}{\eta} < F'(K_2)$$

Appendix C1

This Appendix shows how to get the result in the conditional lending case. The utility function is:

$$U = Y_1 - K_2 + D_1(K_2) + \int_{\underline{A}}^{\frac{D^*}{\eta F(K_2)}} [AF(K_2)(1-\eta)]\,\pi(A)dA +$$

$$+\int_{\frac{D^*}{\eta F(K_2)}}^{\bar{A}(K^*)} \left[AF(K_2) - (D_1(K_2)+D)\right]\pi(A)dA$$

I maximize it with respect to K_2:

$$\frac{\delta U}{\delta K_2} = -1+\eta E(A)F'(K_2)+(1-\eta)F'(K_2)\int_{\underline{A}}^{A^*} A\pi(A)dA+(1-\eta)F(K_2)\frac{D+D_1(K_2)}{\eta F(K_2)}\pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)(...)+$$

$$+F'(K_2)\int_{A^*}^{\bar{A}(K_2)} A\pi(A)dA + F(K_2)\left[\bar{A}(K_2)\pi(\bar{A}(K_2))\bar{A}'(K_2) - \frac{D+D_1(K_2)}{\eta F(K_2)}\pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)(...)\right]+$$

$$-\eta E(A)F'(K_2)\int_{A^*}^{\bar{A}(K_2)} \pi(A)dA - \eta E(A)F(K_2)\left[\pi(\bar{A}(K_2)\bar{A}'(K_2) - \pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)(...)\right] = 0$$

and rearranging I get:

$$F'(K_2)\left[E(A) - \eta\int_{\underline{A}}^{A^*} A\pi(A)dA + \eta E(A)\int_{\underline{A}}^{A^*}\pi(A)dA\right] = 1 - F(K_2)\pi(\bar{A}(K_2))\bar{A}'(K_2)\left[\bar{A}(K_2) - \eta E(A)\right]$$

It is necessary now to make some assumptions for the shock probability distribution. To do that, I consider that the probability of default in case of conditional lending can be written as:

$$\int_{\underline{A}}^{\frac{D+D_1(K_2)}{\eta F(K_2)}} \pi(A)dA$$

or

$$1 - \int_{\frac{D+D_1(K_2)}{\eta F(K_2)}}^{\bar{A}(K_2)} \pi(A)dA$$

Thus, the derivative of these two expressions with respect to K_2 is:

$$\frac{\delta}{\delta K_2} = \pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)\frac{\frac{\delta D_1(K_2)}{\delta K_2}\eta F(K_2) - (D+D_1(K_2))\eta F'(K_2)}{\eta^2 F(K_2)^2} = 0$$

and

$$\frac{\delta}{\delta K_2} = -\left(\pi(\bar{A}(K_2))\bar{A}'(K_2) - \pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)\frac{\frac{\delta D_1(K_2)}{\delta K_2}\eta F(K_2) - (D+D_1(K_2))\eta F'(K_2)}{\eta^2 F(K_2)^2}\right) = 0$$

that must be equal:

$$\pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)\frac{\frac{\delta D_1(K_2)}{\delta K_2}\eta F(K_2) - (D+D_1(K_2))\eta F'(K_2)}{\eta^2 F(K_2)^2} = +$$

$$-\left(\pi(\bar{A}(K_2))\bar{A}'(K_2) - \pi\left(\frac{D+D_1(K_2)}{\eta F(K_2)}\right)\frac{\frac{\delta D_1(K_2)}{\delta K_2}\eta F(K_2) - (D+D_1(K_2))\eta F'(K_2)}{\eta^2 F(K_2)^2}\right)$$

Thus, it must be that: $\pi(\bar{A}(K_2)) = 0$ and/or $\bar{A}'(K_2) = 0$.
Then, coming back to the conditional lending result found before it becomes:

$$F'(K_2)\left[E(A) - \eta \int_{\underline{A}}^{A^*} A\pi(A)dA + \eta E(A) \int_{\underline{A}}^{A^*} \pi(A)dA\right] = 1$$