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Endogenous Technical Change in Alternative Theories of Growth and Distribution

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Abstract

This paper surveys the last two and a half decades of non-neoclassical literature on endogenous technical change and the factor distribution of income. The implications of classical-Marxian and post-Keynesian contributions are compared with neoclassical exogenous and endogenous growth theories. We find the comparison illuminating in several respects: despite the strong differences in the assumptions regarding the substitutability between capital and labor, the role of different classes in society, and whether or not productive factors are fully employed, the various alternative models can be classified in a way that highlights remarkable similarities with their neoclassical counterparts. Both neoclassical and alternative theories of endogenous growth: (i) have shown that long-run growth is sensitive to investment decisions; (ii) rely on a linear spillover from the stock of knowledge to the production of innovations, and (iii) match the Kaldor facts in the long run. The comparison allows to evaluate competing theories by looking at the different channels they emphasize: saving behavior and market structure in the neoclassical theories, as opposed to income distribution, the state of the labor market, and investors' behavior in alternative theories.

Keywords: Endogenous technical change, neoclassical growth, alternative theories, income distribution.

JEL Classification Codes: B50, D33, O33.

1 Introduction

The analysis of the role played by technological change in shaping an economy's growth path is of central importance in classical political economy. In his *Wealth of Nations,* Adam Smith famously emphasized increasing returns and specialization as the main driver of economic progress (Smith,

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1776, I.iii), while in the third volume of *Capital*, Marx focused on the profit-driven motive to innovation in capitalist economies, and the corresponding conflictual nature of labor productivity growth Marx (1867). With the marginalist revolution and its main concern with the allocation of scarce resources over competing needs, technical progress fell out of fashion and was either assumed away or to take place exogenously, as it is the case in the (augmented) Solow (1956) model. Only in the early 1990s a revived interest in the endogenous determinants of technical change allowed neoclassical economists to embed insights by Schumpeter (1942) into dynamic general equilibrium growth models (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). These theories have defined endogenous technical change as: (i) explained within the model rather than assumed to occur; (ii) dependent on preferences, in particular as it pertains to the allocation between current and future consumption, and policy action; (iii) costly to generate, so that the problem of allocating resources to R&D becomes of crucial importance.

Before the advent of endogenous growth, the neoclassical scene was dominated by the Solow growth model and its optimal growth counterpart (Ramsey, 1928; Cass, 1965; Koopmans, 1965). As pointed out by Jones and Romer (2010), one reason for the success of the neoclassical growth model was its ability to match the so-called stylized facts of long run growth highlighted by Kaldor (1961). Among those, here we are mostly concerned with: (a) the constancy of factor shares in the long-run; (b) a purely labor-augmenting profile of technical change; (c) an increasing capital intensity, and (d) the constancy of the output/capital ratio. In order to match the Kaldor facts, the neoclassical theory hinges crucially on the use of a smooth aggregate production function, and the resulting choice of the technique of production which equates factors marginal products to their prices.

The Cambridge capital controversy of the 1960s has warned in a definitive way about the logical shortcomings of neoclassical aggregate capital theory. While the neoclassical prescriptions about factor substitution and the distribution of income hold true in a one-good economy, it is not possible to deduct an aggregate production function with diminishing real marginal product of capital from an economy with heterogeneous capital goods (Garegnani, 1970; Samuelson, 1966). Thus, the Cambridge critique had disruptive implications for the neoclassical theory of growth and distribution. Not surprisingly, at the time when the capital debate was occurring, non-neoclassical economists began to look again at the process of technical change, as opposed to capital/labor substitution, in order to provide an alternative view of distribution based on endogenous technical progress that was compatible with the Kaldor facts, but at the same time would be immune from the pitfalls of the marginalist theory. Key examples of these efforts are the induced innovation hypothesis by Charles Kennedy (Kennedy, 1964), and Nicholas Kaldor's technical progress function (Kaldor, 1957).

Despite neither Kaldor's nor Kennedy's view of technical change ever became mainstream, they laid the foundations for more recent work by economists working in alternative traditions toward modeling the interplay between the factor distribution of income and the evolution of labor productivity over time. This survey is meant to summarize the last two and a half decades of alternative literature on balanced growth models that reject the notion of an aggregate production function with well-defined marginal products, but whose long-run equilibria are consistent with the Kaldor facts. We will distinguish between classical-Marxian and post-Keynesian models, and analyze the implications different model closures have for income distribution and productivity growth. Using these alternative models, we will look at three viewpoints on the evolution of technology: (i) a classical-Marxian technical progress function, where labor productivity growth depends on factor shares; (ii) a Kaldorian technical progress function, in which labor productivity grows in line with capital accumulation, and (iii) a technical progress function that relates labor productivity growth with tightness in the labor market. In addition, we provide an account of recent research on costly innovation in classical models, and of an unbalanced growth model featuring a declining output/capital ratio with a strong Marxian flavor. These broadly categorized alternative theories are compared with neoclassical exogenous and endogenous growth: despite a number of strong differences, there are striking similarities between the approaches.

The paper is organized as follows. Section 2 reviews exogenous and endogenous mainstream growth theories in order to provide a benchmark for comparison. Section 3 looks at three different specifications of endogenous technical change used in the alternative literature, while Sections 4 and 5 study their implications for different model closures: classical supply side closures with either exogenous distribution or labor supply, *vis à vis* two post-Keynesian accumulation closures (Kaleckian and Kaldorian respectively). Section 6 analyzes the empirically relevant unbalanced growth scenario involving decreasing capital productivity and a falling rate of profit (the so-called Marx-biased technical change case), while Section 7 reviews contributions that introduce mainstream endogenous growth insights in an otherwise classical model. Section 8 concludes.

2 Neoclassical Growth Theory

2.1 Common Elements

In what follows, we provide a short summary of the mainstream growth theories of technical change, both exogenous and endogenous, by focusing on their implications for economic growth and income distribution. The goal is to highlight the distinctive features that allow for a comparison with the alternative growth and distribution theories that are at the center of this paper.

The neoclassical (or marginalist) theory of output and distribution is based on technology, consumer preferences, and endowments of productive factors such as capital and labor. This structure produces a few foundational elements common to both exogenous and endogenous growth. First, all models presuppose Say's law thus omitting any considerations about the role of aggregate demand in the growth process: growth is determined by supply factors alone. The existence of a continuum of techniques of production ensures that it is possible to substitute one factor to another until their endowments are fully employed. Excess supply of one input would produce a reduction in its price, thus favoring the adoption of a technique of production which employs the factor more intensively. Second, income distribution is determined by the relative scarcity of the productive factors. The interaction between technology and factor endowments determines the equilibrium marginal product of each factor, which equals its rate of remuneration via profit-maximization by firms. Due to diminishing marginal products, relative factor prices decrease with the relative factors supply. Third, the society's preferences with regards to consumption and savings affect the endowment of the accumulable factors of production, and therefore output growth and income distribution. Fourth, the economic environment is typically (although not universally) modeled with the aid of a representative agent. The typical agent earns a salary as worker and receives interest income being the owner of capital assets. Thus, there is no class distinction in the economy: incomes are differentiated according to their source, not by the social class to which they accrue.

2.2 Exogenous Growth

The Solow (1956) growth model, and its Ramsey-Cass-Koopmans counterpart featuring an endogenous saving rate (Ramsey, 1928; Cass, 1965; Koopmans, 1965), see technical change as purely exogenous. In fact, under the assumption of perfectly competitive goods and factor markets as well as marginal productivity pricing of capital and labor, neoclassical growth *requires* technical change to be generated outside the model: because the aggregate production function is linearly homogeneous in capital stock K and labor L, if both factors of production are paid their marginal product there are no resources left to innovate. This follows from Euler's theorem: if, for a given level of technology $\bar{A}$, output Y is produced according to a constant returns to scale and twice continuously differentiable function of capital and labor $F(K, L, \bar{A})$, Euler's theorem implies that $F_K K + F_L L = Y$, where F_i is the marginal product of factor i. Hence, remunerating capital and labor takes up the entire national product, and no resources are left to finance the production of technology-improving innovations. Accordingly, the growth rate of technology $\dot{A}/A \equiv g_A$ is necessarily exogenous. If in order to focus on balanced growth we assume that technical progress is labor augmenting (Uzawa, 1961), we can re-write the production function as $F(K, AL)$, where AL is a measure of labor in efficiency units, or effective workers. Let $k \equiv K/(AL)$. Then, output per effective worker is $y \equiv Y/(AL) = f(k)$. Assuming no depreciation and constant population growth at a rate $n > 0$, the steady state of the Solow model solves

$$\frac{f(k_{ss})}{k_{ss}} = \frac{g_A + n}{s}. \tag{1}$$

The left-hand side of equation (1) features the constant long-run output/capital ratio, which increases in the growth rate of the effective labor force $g_A + n$ and decreases in the saving rate.

While growth is exogenous, income distribution is endogenous. The key variable determining the steady state distribution of income is the long run capital intensity (in efficiency units) k_{ss}, as it regulates both factor prices and factor shares. Under marginal productivity pricing, the interest rate r equals the marginal product of capital, and in the long run $r = F_K = f'(k_{ss})$. Factor shares, on the other hand, coincide with the respective output elasticities: if we let ω be the wage share in national income, its long-run value is $\omega_{ss} = 1 - f'(k_{ss})k_{ss}/f(k_{ss})$, where $f'(k_{ss})k_{ss}/f(k_{ss})$ is the elasticity of output with respect to capital. Therefore, all relevant measures of income distribution are endogenous as they depend on the capital/labor ratio, save for the special (though admittedly popular) Cobb-Douglas case where output elasticities are parametrically constant throughout the entire growth path. The saving rate and the exogenous growth rate of technology influence the long-run distribution of income by affecting the steady state capital intensity in opposite ways. The direction of such influence is regulated by the elasticity of substitution between factors of production σ, defined as the percentage increase in the capital/labor ratio due to a percentage increase in the ratio of marginal products (in turn equal to the wage/interest rate ratio). When $\sigma < 1$ (> 1), an increase in the wage/interest rate ratio produces a less (more) than proportional boost in the capital/output ratio, so that the wage share increases (decreases). The long-run capital intensity increases in the saving rate: therefore, an increase in s raises (lowers) the long-run wage share if $\sigma < 1$ (> 1). The opposite is true for the growth rate of technology, which is inversely related with the steady state capital/labor ratio. Finally, when $\sigma = 1$ we are in the Cobb-Douglas case, and income shares are equal to the constant elasticities of output with respect to capital and labor.

2.3 Endogenous Growth

Starting in the early 1990s, mainstream economists have investigated the role of ideas in generating increasing returns that allow for sustained economic growth in the long run. The endogenous growth models that followed, either based on increasing product variety (Romer, 1990) or increasing product quality (Aghion and Howitt, 1992; Grossman and Helpman, 1991), produce endogenous technical change that is fully explained within the model, and is affected by saving behavior and policy action. The enterprise was made possible by abandoning perfect competition in favor of monopolistic competition. New ideas are invented in a competitive R&D sector, but once discovered they confer a patent to innovators, who can then earn monopolistic profits that justify the cost of R&D investment.

A version of the Romer (1990) model that allows to appreciate the endogenous nature of technical change is the following. Consider a three-sector economy (final sector, intermediate goods sector, and R&D sector) with a constant, fully employed labor force. The final good is produced under perfectly competitive conditions, according to

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha di, \tag{2}$$

where L_Y is the number of manufacturing workers, x_i denotes the intermediate durable input i rented in the production of the final good, A is a measure of product variety or the number of existing intermediate input in the economy, and $\alpha \in (0,1)$ is an elasticity parameter. One unit of raw capital can be transformed in one unit of any intermediate input; hence the total amount of intermediate input is equal to the economy's capital stock: $\int_0^A x_i di = K$. As it will be clear just below, the symmetric structure of the model ensures that all intermediate inputs are demanded in the same amount, therefore $x_i = x$, $K = Ax$, and $Y = AL_Y^{1-\alpha}x^\alpha$. This last way of writing the production function shows that output is linear in A, the number of intermediate inputs. Alternatively, one can see that in the aggregate production function the number of varieties plays the role of labor-augmenting technology. In fact, since $x = K/A, Y = (AL_Y)^{1-\alpha}K^{\alpha}$. Being the marginal product of each intermediate input independent of all the others, capital accumulation can escape decreasing returns to scale if it occurs through an increase in the number of intermediate inputs rather than in the amount of each input produced.

Firms in the intermediate goods sector acquire from the R&D sector an infinitely-lived patent on the new variety of input they produce. Therefore, they operate under monopolistic conditions. Each firm chooses the amount of its intermediate good to supply to maximize profits, given a downward sloping inverse demand curve for their product $p(x)$, the one-to-one production technology, and the cost of capital r. The demand curve equals the marginal product of intermediate goods: $p(x) = \partial Y/\partial x = \alpha L_Y^{1-\alpha}x^{\alpha-1}$, and profits to be maximized are $\pi = p(x)x - rx$. The solution is $x = L_Y(\alpha^2/r)^{\frac{1}{1-\alpha}}$, and monopolists earn strictly positive profits, equal across all sectors:

$$\pi = \left(\frac{1-\alpha}{\alpha}\right)\alpha^{2/(1-\alpha)}r^{\alpha/(\alpha-1)}L_Y = \left(\frac{1-\alpha}{\alpha}\right)rx.$$

Monopolistic profits have a fundamental role in generating endogenous technical change, as they provide the resources needed to finance the cost of innovation.

Technological advancements, on the other hand, are developed in a competitive R&D sector. Here, L_A workers are employed to produce new ideas according to $\dot{A} = \lambda A L_A$, which means that the productivity of each R&D worker, λA, is linear in the stock of the existing ideas. Labor moves freely between R&D and final production, and all workers are paid the same wage, equal to the marginal product of labor in the final sector: $w = (1-\alpha)Y/L_Y$. If the allocation of workers between the two sectors is constant, and $\rho \in (0, 1)$ is the fraction of workers in R&D, to be determined within the model, the growth rate is:

$$g_A = \lambda\rho L, \lambda > 0 \tag{3}$$

which makes it clear that economic policy aimed at increasing the number of scientists has permanent growth effects. In fact, the allocation of workers between the R&D sector and the final good sector represents the economy's choice between current consumption versus long-run growth, and it may be affected by policy action by means of tax and subsidies to either sector.

If the research sector is perfectly competitive with free entry, the flow of profits in R&D must be zero. The value of an innovation is given by π/r, the discounted value of profit flow at the market interest rate r. Since one worker produces λA ideas in the unit time, an R&D firm employing L_A workers is faced with the following zero-profit condition:

$$\frac{\pi}{r}\lambda A L_A - wL_A = 0, \text{or } r = \frac{\lambda A\pi}{w}. \tag{4}$$

We can now solve for the rate of return as a simple function of the elasticity of demand faced by the monopolist α, the R&D productivity parameter λ, and the number of workers in the final good sector $(1-\rho)L$ as follows:

$$r = \alpha\lambda L_Y = \alpha\lambda(1-\rho)L$$

To see the effect of the saving rate on long-run growth observe that, with a constant saving rate $s \in (0,1)$ for simplicity, the growth rate of capital is

$$g_K = s\frac{Y}{K} = s\left(\frac{L_Y}{x}\right)^{1-\alpha} = s\frac{r}{\alpha^2} = \frac{s\lambda(1-\rho)L}{\alpha}.$$

Next, the balanced growth condition $g_X = g_A$ yields the share of workers in the R&D sector as

$$\rho(s) = \frac{s}{s+\alpha}. \tag{5}$$

A higher saving rate shifts the allocation of workers in favor of the R&D sector, as society prefers to improve future technology over current consumption. Since the growth rate of technology is linear in the number of researchers, a higher saving rate determines a permanent increase in the growth rate of the economy, in contrast with the Solow model.

Consider also the implications of this class of models for income distribution. The labor share in output is endogenous as it depends on the fraction of workers in the R&D sector. In fact, using the wage equation above together with the labor market clearing condition, we have:

$$\omega = \frac{1-\alpha}{1-\rho(s)}. \tag{6}$$

A combined look at equations (5) and (6) shows that the wage share increases in the saving rate. The wage rate is determined in the final good sector as the marginal product of labor, which in turn decreases in the share of workers in that sector: a higher saving rate shifts the composition of the labor force toward R&D, thus making final good-producing workers more productive and increasing wages. Notice finally that the output/capital ratio is inversely related to the saving rate, just like in the neoclassical exogenous growth model. In fact, $Y/K = \lambda(1-\rho(s))L/\alpha$, decreasing in the saving rate.

An insightful paper by Kurz and Salvadori (1998) has shown that income distribution in early neoclassical endogenous growth models depends on technology and profit maximization only, and it is therefore exogenous. Our account is not inconsistent with their conclusion, once a fundamental difference in the assumptions of the models under consideration is singled out. In our analysis, the factor of production responsible for sustained growth (ideas) requires labor, a scarce input, in order to be produced. In the models considered by Kurz and Salvadori (1998), on the other hand, no original input is involved in the accumulation of the factors responsible for endogenous growth, whether these be physical capital, human capital or ideas. The difference is thus reconciled in our framework by simply assuming that innovations are produced using foregone consumption (in terms of final output) rather than labor; the latter is fully employed in producing the final good. Equations (2) and (3) become $Y = L^{1-\alpha}\int_0^A x_i^\alpha di$ and $\dot{A} = \lambda\rho'(Y-X)$, where ρ' is the share of disposable income that finances the R&D effort. The marginal product of labor is now $F_L = w = (1-\alpha)Y/L$, so that the wage share $\omega = 1 - \alpha$ is independent of the saving rate.

A crucial implication of the growth equation (3) is that the growth rate increases with the size of the labor force. This feature is known as the *scale effect* property of first-generation endogenous growth models, and with population growing at a rate $n > 0$ implies an explosive growth rate. The scale effect property was questioned on empirical grounds by Jones (1995): he proposed a specification for the growth rate of varieties featuring a less-than proportional spillover from past discoveries: with $\dot{A} = \lambda\rho L A^{\varphi}, \varphi \in (0,1)$, the growth rate of ideas is:

$$g_A = \lambda\rho L A^{\varphi-1}. \tag{7}$$

Balanced growth requires a constant growth rate. Differentiating (7) with respect to time and imposing $\dot{g}_A = 0$ yields

$$g_A = \frac{n}{1-\varphi} \tag{8}$$

Economic growth is therefore explained within the model, but is independent of savings and policy. For this reason, this class of models is commonly referred to as *semi-endogenous.* In terms of income distribution, the model basically reproduces equation (6): the labor share is endogenous, and increasing in the saving rate. The semi-endogenous neoclassical growth model provides a first element of comparison with alternative theories: we will show below that this model displays strong similarities with Kaldorian growth models with exogenous labor supply.

As a final observation, the comparison of endogenous vs. semi-endogenous growth models highlights the importance of the spillover generated by past discoveries in the production of new ideas, or alternatively the lack of robustness of the assumptions required to generate endogenous growth (Solow, 1994, 2007). With constant returns to ideas (the reproducible factor), growth is endogenous: thriftier consumers or innovation subsidies that increase the percentage of scientists in the labor force will have permanent effects on the growth rate. With diminishing returns, neither savings nor innovation subsidies have permanent effects on growth. In the language of dynamical system theory, endogenous growth models are *structurally unstable*: their conclusions are not robust to slight modifications in the elasticity parameter in the innovation function.

3 Alternative Theories of Growth and Distribution

3.1 Common Elements

Even though different contributions have emphasized different aspects of the interplay between growth and distribution in the long run, there are a few common elements that span almost universally throughout the non-mainstream literature. A first feature is the use of a Leontief aggregate production function combining fixed proportions of capital K and labor L in producing output Y:

$$Y = min[uBK, AL], \tag{9}$$

where B denotes the output/capital ratio, A is labor productivity, and $u \equiv Y/Y^p$ is a measure of capacity utilization. Such production technology is not always explicitly assumed but it is functional as a critical tool toward mainstream economics, in that: (a) it implies a rejection of marginal productivity theory, as marginal products are not defined, and (b) it allows for less than full employment of both capital and labor. In fact, because factor demands are inelastic to factor prices, there is no market-based equilibrating mechanism toward full utilization of all factors of productions. Thus, even when there is full utilization of capital –that is, when $u = 1$– the production technology allows for structural unemployment of labor. When effective demand is not strong enough to ensure full capacity utilization, that is when $u < 1$, both capital and labor are unemployed.

A second, distinctive element of non-conventional theories is their focus on class as a defining feature of capitalist economies. The distinction between workers on the one hand, and owners of the capital goods (capitalists) on the other, brings the functional income distribution front and center in the analysis. Given the production technology, the inverse relationship between wages and profits is represented by the distributive curve:

$$r = uB(1 - \omega). \tag{10}$$

where $wL/Y = w/A \equiv \omega$ is the share of labor in production. The class distinction of society pertains also to different saving behavior: it is assumed that workers and capitalists have different propensities to save $s_w < s_\pi \equiv s$. Here, we use a simplified version of differential savings by assuming $s_w = 0$ throughout the analysis.

A third common feature in the non-mainstream literature is that technical change is (with some exceptions discussed in Section 7) *costless*. In the neoclassical endogenous growth literature, the production of new technologies requires to allocate scarce resources to R&D investment, and is therefore costly. On the contrary, the problem of resource allocation toward the financing of innovation technology is seldom addressed in alternative treatments of economic growth, and technical change is basically explained within the model as an externality, whether it occurs through learning by doing or as a function of income distribution and employment, as it will be clear just below.

3.2 Approaches to Technical Change

In general, technological change can be represented by changes in the productivity of both labor and capital. However, balanced growth with constant returns to scale in production requires $g_B \equiv \dot{B}/B = 0$, or constant capital productivity in the long run, as shown by Uzawa (1961), and recently re-emphasized by others (Schlicht, 2006; Jones and Scrimgeour, 2008; Irmen, 2016). For this reason, most models (with the exceptions discussed in Sections 4.3 and 6) simply assume a constant output/capital ratio, and deal only with the growth rate of labor productivity g_A.

We will then investigate three routes to model improvements in labor productivity. The first one is known as Kaldor-Verdoorn law, and it states that technical change is directly related either to capital accumulation or to output growth. The law is based on empirical work by Verdoorn (1949) and its theoretical formalization proposed by Kaldor (1957) by means of a technical progress function. The basic idea, however, has been a mainstay in the understanding of economic development since the inception of political economy, when Adam Smith linked productivity growth to the division of labor, in turn limited by the size of the market (Smith, 1776, I.iii). Later, Marshall (1920, IV), Young (1928) and Arrow (1962) all related labor productivity to the economy's scale of production and capital stock through the concepts of external economies of scale, macroeconomic increasing returns, and learning by doing. We will adopt two linear versions of the law, where productivity growth depends either on the growth rate of absolute capital or on the growth of capital per worker. Denoting the aggregate growth rate of capital stock by g_K and the growth rate of labor demand by g_L, the technical progress function in absolute terms can be written as:

$$g_A = \varphi_0 + \varphi_1 g_K, \tag{11}$$

and in per capita terms as:

$$g_A = \varphi_0 + \varphi_1 (g_K - g_L), \tag{12}$$

with φ_0, φ_1 positive constants. The latter specification corresponds to a log-linear representation of the level of labor productivity: $A = A_0(K/L)^{\varphi_1}$ with φ_0 denoting the growth rate of the scale parameter A_0.

A second option consists in assuming that labor-saving technical change depends on the labor share. This relation is intuitive if one thinks about a firm's quest for cost-minimization, and is founded in the classical-Marxian analysis of the choice of technique. New techniques of production are adopted only if they do not decrease the profit rate at the given real wage (Okishio, 1961); when the wage share (i.e. the unit labor cost of the individual firms) rises, an increase in labor productivity is necessary if the firm does not want to lower its rate of profit. Thus, labor-saving innovation is a way to re-establish profitability in the face of rising labor costs. Some of the contributions belonging to this tradition allow for variable capital productivity, and we will analyze them separately in Sections 4.3 and 6. For now, it is enough to focus on a constant output/capital ratio while assuming a direct relationship running from the wage share to the growth rate of labor productivity:

$$g_A = f(\omega) \tag{13}$$

as in Taylor (1991) and Dutt (2013).

A third strand of recent literature (Dutt, 2006; Palley, 2012; Sasaki, 2010; Setterfield, 2013a) has looked at labor market tightness –measured by the employment rate– as a potential channel for technical change. Shortages of labor would push firms to adopt innovations that save on labor requirements. From a conceptual point of view, the approach is similar to the one just discussed, because it builds on the implicit assumption that a tight labor market reduces firms' profit margins. However, we will see in Sections 5.1 and 5.2 that the focus on the labor market rather than on distribution is favored within the Keynesian demand-led growth framework, as it is instrumental in reconciling actual and potential growth rates. Given the exogenous labor supply N, we define $e \equiv$

L/N the employment rate in the economy. We can therefore represent this approach to technical change as

$$g_A = h(e),\ h' > 0. \tag{14}$$

Observe that equations (18), (17) and (14) all arise from a general specification of the kind $\dot{A} = G(\cdot)A$, which exhibits the linear spillover of existing technologies that we discussed regarding the neoclassical endogenous growth case.

Finally, in what follows we will discuss two cases in which capital productivity is not constant. The output/capital ratio might vary along the transitional dynamics toward the balanced growth path, as the model evolves toward a constant long run output/capital ratio: such would be the case with induced technical change, which we study in Section 4.3.1. Alternatively, the rejection of balanced growth has led authors to focus on the case of capital-using technical change involving negative capital productivity growth. Combined with a classical closure, a falling output/capital ratio determines a falling rate of profit: for this reason, such a pattern is typically referred to as Marx-biased technical change (Foley and Michl, 1999, Chapter 7), which is analyzed in Section 6.

3.3 Balanced Growth with Endogenous Technical Change

Roy Harrod's essay on dynamic economics (Harrod, 1939) is conventionally seen as the beginning of modern growth theory. A stylized account of his contribution would define three different growth rates: the actual growth rate g_K^i, or the ratio of investment to capital stock; the 'warranted' rate g_K^s, that is the ratio of savings to capital; and the 'natural' rate g^p, given by the sum of population growth and exogenous labor productivity growth. The Harrodian analysis devises two problems. First, there are no self-correcting mechanisms capable of dampening deviations of the actual rate from the warranted rate: this is known as the Harrodian instability problem.[1] Second, full employment is the exception rather than the norm: the economy needs to expand at its natural rate to keep a steady employment rate, but nothing ensures this will happen because the determinants of the warranted and natural rate are unrelated. To some extent, the development of the theory of economic growth can be seen as an attempt to address these two issues. Our focus here is to investigate how the introduction of endogenous technical change affects the solution that different economic traditions have given to the second Harrodian problem.

Given the analytical framework described in Section (3.1), the warranted growth rate is

$$g_K^s = S/K = sr = suB(1 - \omega), \tag{15}$$

[1] Somewhat ironically, neoclassical endogenous growth models are not immune from Harrodian instability. In fact, their balanced growth equilibrium has 'no transitional dynamics' (which means that is unstable), because it is reached through instantaneous jumps in the forward-looking variable (consumption) onto the balanced growth path. Such jumps occur through rational expectations, and are not possible in the original Harrod (1939) model because of the constant saving rate assumption. The irony is that the Solow (1956) model solved the Harrodian instability problem through diminishing returns to the reproducible factor. Endogenous growth overcomes diminishing returns in reproducible factors, bringing back instability into the picture. The skepticism expressed by Solow (1994, 2007) is then not at all surprising.

while the natural rate of growth is

$$g^p = n + g_A.$$

Notice that g^p also represents a rate of growth of potential output, since it is a measure of the overall growth in factors supply. The actual rate of growth of capital stock is theory-specific: below, we distinguish between the classical tradition on the one hand, and post-Keynesian traditions on the other, in turn drawing a difference between Kaleckian models and Kaldorian models

The classical tradition is founded on the acceptance of Say's law, the notion that supply creates its own demand at the aggregate level. This principle is satisfied when all savings are automatically invested in the accumulation of capital, and it amounts to impose full utilization of capital stock: $u = 1$. In fact, an independent investment function is absent, and we can directly impose

$$g_K^i \equiv g_K^s. \tag{16}$$

A crucial element of post-Keynesian economics, instead, is an investment demand function that is independent of saving behavior. In the (neo-) Kaleckian tradition, investment depends on utilization as a proxy of aggregate demand (an accelerator effect), and a measure of profitability. The latter determinant of investment is justified through the fact that a firm's current profits provide a source of internal funds that allow to accumulate capital without resorting to credit markets, and are also an indication of future profitability. While early authors have used a specification that involved the profit rate as the main determinant of investment demand (Taylor, 1991), after Bhaduri and Marglin (1990) neo-Kaleckian economists have looked at a relationship between investment and the profit share that can be written as follows:

$$g_K^i = \gamma + \eta_0 u + \eta_1 (1 - \omega). \tag{17}$$

Finally, the Kaldorian tradition emphasizes exports as the ultimate source of autonomous aggregate demand. Since labor productivity growth improves an economy's competitiveness, thus providing access to a larger share of global demand, we can assume that investment rises with labor productivity growth :

$$g_K^i = \gamma + \lambda g_A. \tag{18}$$

In fact, (18) can be seen as a reduced form for investment demand in an open economy, where the coefficient λ captures trade-related factors such as the foreign trade multiplier and the price-elasticity of exports (see Setterfield, 2013b, for a derivation). We keep the intercept γ to facilitate comparisons to with the Kaleckian framework. In both (17) and (18), the parameter γ can be interpreted as the autonomous growth rate of investment demand –Keynes' *animal spirits*. The equality $g_K^i = g_K^s$ defines the short-run equilibrium growth rate g^*. On the other hand, balanced growth requires that $g^* = g^p$, or we would face the second Harrodian problem with cumulative disequilibrium in the labor market. Below, we explore the implications for growth and distribution of different specification of g_A for classical and post-Keynesian economics.

4 Classical Closures

As already mentioned, a common element in the contributions falling within the classical tradition is the acceptance of Say's law, which amounts to impose full utilization of installed capacity in the general framework. The main difference pertains to models featuring a distributive closure and endogenous growth, as opposed to models featuring an exogenous labor supply closure and an endogenous determination of income distribution.

4.1 Distributive Closure

The classical-Marxian tradition has emphasized the role of the reserve army of labor in keeping the labor share in check. The most notable example in development economics is that of a dual economy with a large rural sector providing a basically unlimited pool of labor from which a small but growing manufacturing sector can draw (Lewis, 1954). A dual economy is not labor-constrained: rather, it is capital stock to be the limiting factor for growth. Accordingly, the classical closure amounts to impose a perfectly elastic labor supply at the going wage share. Income distribution is exogenous, and the wage share is fixed at its *conventional* value (Foley and Michl, 1999):

$$\omega = \bar{\omega}. \tag{19}$$

On the other hand, capital stock is assumed to be utilized at its normal rate: $u = 1$. This model delivers endogenous growth even without technical change. In fact, the growth rate is simply

$$g^* = sB(1 - \bar{\omega}). \tag{20}$$

Because there is no constraint arising from the size of the labor force, the rate at which the economy grows is governed by capital accumulation only. On the other hand, labor supply is endogenous and it accommodates labor demand: $\dot{N}/N = g_L$. Notice the similarity of the steady state of this model with the Romer model using foregone consumption as an input to R&D: both models deliver an endogenous growth rate that increases in the saving rate, coupled with an exogenous distribution.

In this context, endogenous technical change adds very little to the analysis, as shown in Dutt (2011) and Dutt (2013); in fact, the role of technical change is simply to determine the growth of employment through the long-run condition $g^* = g^p = g_L + g_A$. Assume for example that technology evolves according to (12): labor productivity growth is $g_A = \varphi_0/(1 - \varphi_1)$, and is therefore semi-endogenous; while the employment rate satisfies $g_L = sB(1 - \bar{\omega}) - \varphi_0/(1 - \varphi_1)$. Similarly, if we model g_A through the Marxian motive described in (13), we find $g_L = sB(1 - \bar{\omega}) - f(\bar{\omega})$. In both cases, a higher saving rate as well as a higher profit share increase the growth rate of employment, while improvements in innovation technology reduce it. The third route to technical change, as modeled in (14) is incompatible with this framework: with unlimited labor supply, the employment rate is undefined.

4.2 Labor Supply Closure

The classical model of growth has also been investigated under the assumption of exogenous labor supply (Pasinetti, 1974; Foley and Michl, 1999, Chapter 6). The important question that this framework is called to answer is how adjustments in income distribution can maintain a constant employment rate in the absence of capital/labor substitution. When technical change is exogenous, the long-run equilibrium condition $g^* = g^p$ yields the Goodwin (1967) steady state income distribution $1 - \omega_{ss} = (n + g_A)/sB$, which would be same in the Pasinetti (1962) model.[2] The saving rate and labor productivity growth have opposite effects on the wage share. An increase in the saving propensity s lowers the profit share and the profit rate. The reason is that higher savings translate into higher long-run investment and employment growth. Because the employment rate has to remain constant, given an exogenous growth rate of the labor supply a faster pace of accumulation results in pressure on wages relative to labor productivity, so that the wage share increases. Higher labor productivity growth, on the contrary, lowers the economy's labor requirements and the wage share falls as a consequence.

Let us now investigate the implications of introducing endogenous technical change. When g_A follows (12), balanced growth yields

$$1 - \omega_{ss} = \frac{n}{sB} + \frac{\varphi_0}{sB(1 - \varphi_1)}, \tag{21}$$

and

$$g_A = \frac{\varphi_0}{1 - \varphi_1}. \tag{22}$$

Not much changes in terms of income distribution: the labor share increases in the saving rate, and the only difference is that the role of exogenous labor productivity growth is played by the technological parameters of the technical progress function. Long-run technical change, on the other hand, is of the semi-endogenous variety. It is explained within the model, but it is independent of savings (see Taylor, 2004, Chapter 5). Strikingly, this model delivers implications about long-run growth and distribution that are virtually identical to those found in the Jones (1995) model. In fact, as we will discuss in (5.2) this is also the same steady state of a closed-economy Kaldor growth model with endogenous technical change.

When labor productivity grows in Marxian fashion as in (13), income distribution still needs to

[2] In his seminal paper, Pasinetti (1962) has shown that under $s_\pi > s_w > 0$ the only savings that matters for long-run distribution occurs out of capitalists' income only, a result known as 'Pasinetti theorem'. Michl (2009) has proven a contemporary version of the model with savings arising from optimizing behavior.

adjust to satisfy the long run balance $g^* = g^p$. Accordingly, ω_{ss} solves

$$sB(1 - \omega_{ss}) = f(\omega_{ss}) + n. \tag{23}$$

The effect of the saving preferences on income distribution is the same as before: total differentiation of (23) shows $d\omega_{ss}/ds > 0$. A higher saving rate increases capital accumulation; maintaining the balance in the labor market requires either a reduction in the profit share or an increase in labor productivity, which is also achieved through a higher wage share. On the other hand, technical change and growth are fully endogenous as $g_A = f[\omega_{ss}(s)]$: a higher wage share following the increase in the saving rate has a positive effect on labor productivity growth. A comparison with the exogenous distribution model is instructive. With a conventionally determined wage share, capital accumulation is never constrained by labor supply. Conversely, the labor supply poses a binding constraint to capital accumulation in this model, but such constraint is loosened by the fact that investment can increase the growth rate of labor productivity through the distributional channel. We are not aware of contributions that investigated this result; Dutt (2013) studied this specification in the classical model of growth, but only with the conventional wage share assumption.

Finally, we can explore the implications of assuming (14), even though this specification of technical change has not been implemented in the literature. The balanced growth condition becomes

$$sB(1 - \omega_{ss}) = h(e_{ss}) + n, \tag{24}$$

and it provides an equilibrium locus in the space of income distribution and employment rate. In order to pin down equilibrium values, we can borrow from Goodwin (1967) and assume that real wages grow with the employment rate, say $g_w = m(e), m' > 0$, as a tighter labor market strengthens workers' bargaining power. A constant wage share in the long run requires wages and labor productivity to grow at the same rate. Therefore, the condition $m(e_{ss}) = h(e_{ss})$ fixes the equilibrium employment rate and productivity growth, while the equilibrium wage share follows from (24). Employment and productivity are independent of the saving rate which, similarly to the exogenous productivity growth case, has a positive effect on the wage share.

Although this model has not been studied in the literature, we can take the analysis one step further and introduce an explicit policy variable. Assume that real wage growth is also a function of labor market institutions z, for example the degree of employment protection. If $g_w = m(e, z)$, labor market institutions affect the growth rate of labor productivity, as well as income distribution, through their effect on the equilibrium employment rate. Define z in such a way that $m_z > 0$: then, the effect of a change in z on equilibrium employment and distribution depends on the sign of the partial derivative $m'(e_{ss}, z) - h'(e_{ss})$. If the sign is positive, that is if wages are more responsive than productivity to the employment rate, an increase in labor market protection lowers employment and productivity while raising the wage share. Vice versa, if the sign is negative an increase in z has a positive effect on equilibrium employment but an adverse effect on the labor share. Either way, workers face a trade-off between employment and productivity on one end, and the wage share on the other. Such trade-off is in contrast with the steady-state implications of the Goodwin (1967) model, where an increase in employment protection would reduce employment but would have no impact on income distribution.

4.3 The Induced Innovation Hypothesis

The dissatisfaction with the exogenous nature of technical change in the neoclassical growth model led some scholars to consider the microeconomic choice of factor-augmenting technologies made by firms guided by profit maximization. In particular, while it became clear very soon that balanced growth requires technical change to take the pure labor-augmenting form, the economic rationale for such a biased pattern of technology was not as well understood.

Taking up an old insight by Hicks (1932), who argued that profit-maximizing firms would seek to augment the productivity of the factor of production whose share in total costs increases, Kennedy (1964) postulated the existence of an *innovation possibility frontier* (IPF). The IPF inversely relates the attainable growth rate of labor productivity to the growth rate of capital productivity: $g_B = \phi(g_A), \phi' < 0, \phi'' < 0$. The strict concavity of the IPF captures a notion of increasing complexity in the trade-off between labor-augmenting and capital-augmenting blueprints. The *induced innovation hypothesis* is Kennedy's idea that firms choose, in myopic fashion, a profile of technical change (g_A, g_B) so as to maximize the rate of change in unit cost-reduction $\omega g_A + (1-\omega)g_B$ under the constraint given by the IPF (Funk, 2002). The result of this program is that the growth rate of labor (capital) productivity becomes an increasing function of the wage (profit) share: $g_A = f(\omega), f' > 0$. This expression is formally identical to equation (13), but it does not assume constant capital productivity. The microeconomic appeal of the induced innovation hypothesis has led to a renewed interest to this theory in recent years (Foley, 2003; Julius, 2005; Rada, 2012; Tavani, 2012, 2013; Zamparelli, 2015).

Once the induced innovation hypothesis is adopted, the relation between productivity growth and income distribution in the classical growth model with exogenous labor supply changes dramatically. In its simplest form, the steady state of such classical model consists of the following equations:

$$g_B = \phi(g_A) = \phi[f(\omega_{ss})] = 0 \tag{25}$$

$$g_B + sB_{ss}(1-\omega_{ss}) = f(\omega_{ss}) + n \tag{26}$$

In balanced growth, the output/capital ratio B has to remain constant: therefore, $g_B = \phi[f(\omega_{ss})] = 0$, which solves for a unique value of the labor share in the long run: $\omega_{ss} = f^{-1}[\phi^{-1}(0)]$. The long-run labor productivity growth rate is then determined through the IPF: $g_A = \phi^{-1}(0)$. Finally, once the wage share is found, the balanced growth condition yields the long-run level of capital productivity as

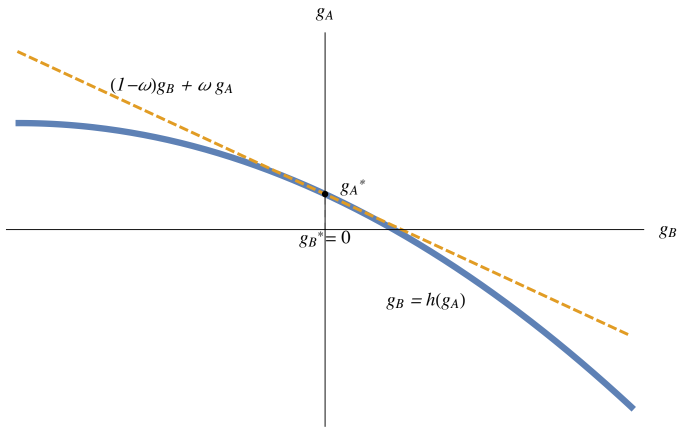

Figure 1: The induced innovation hypothesis.

$$B_{ss} = \frac{f(\omega_{ss}) + n}{s(1-\omega_{ss})}. \tag{27}$$

Important features of the induced innovation model are that: (a) factor shares *adjust* over time in order to ensure a labor-augmenting profile of technical change in the long run, and (b) income distribution depends only on the shape of the IPF. These results are substantially different from the classical model with constant capital productivity, and from the neoclassical model. We have shown how the saving rate in the classical model with endogenous labor productivity growth is a crucial determinant of income distribution, and even on per capita growth under the specification (13) of technical change. We have also discussed in Section 2 the influence of the saving rate on income distribution in neoclassical theory. Under the induced innovation hypothesis, on the contrary, the saving rate has no influence on income distribution and growth; however, similarly to the neoclassical case, it does affect the long-run level of capital productivity. Consider in fact an increase in the saving rate in equation (26), which puts pressure on the accumulation in the left hand side. However, the position and shape of the IPF has not changed: the labor share is fixed, and so is the labor force growth rate. Therefore, the increase in savings must be counteracted in a decrease in the output/capital ratio, as it is clear from equation (27). Figure 1 displays Kennedy's IPF and the long-run profile of technical change in this model.

In a recent paper, Schlicht (2016) has emphasized the connections between the innovation possibility frontier and Kaldor's technical progress function; in fact, Kennedy (1964, 547n) already did notice that "if the technical progress function is known, the innovation possibility function can be derived from it". However, while the two theories are isomorphic in terms of their representation of technology they have different steady state implications regarding income distribution, as a comparison of (21) and (25) shows.

The induced innovation hypothesis has also been implemented in the neoclassical framework beginning with Drandakis and Phelps (1965); Nordhaus (1967); Samuelson (1965); von Weizsacker (1966). Strengths and weaknesses of these models are illustrated in Brugger and Geherke (2016).

4.3.1 Dynamics of Employment and the Output/Capital Ratio

Incorporating the induced innovation hypothesis in the classical Goodwin (1967) growth cycle model highlights the adjustment process toward a constant output/capital ratio, as well as constant income shares and employment rate, in the long run. The most important implication of this model is the disappearance of the perpetual Goodwin cycles as the economy actually reaches the steady state, instead of fluctuating permanently around it (Shah and Desai, 1981). However, the direction of adjustment is not monotonic: Goodwin cycles typically persist in the short-to-medium run under standard parametrizations. The model adds the employment rate $e = BK/(AN)$ as a dynamic variable, and makes use as before of a real-wage Phillips curve so that an increase in employment feeds into higher real wage growth: $g_w = m(e)$ in its simplest form. The model is composed of the following three differential equations (Foley, 2003; Julius, 2005):

$$g_B = \phi[f(\omega)] \tag{28}$$

$$g_\omega = m(e) - f(\omega) \tag{29}$$

$$g_e = \phi[f(\omega)] + sB(1-\omega) - f(\omega) - n \tag{30}$$

In steady state, setting as above $g_B = 0$ solves for the long-run labor share ω_{ss}, which is once again determined by the IPF only. Such long-run value for income distribution can be used to solve for the long-run employment rate as $e_{ss} = m^{-1}[f(\omega_{ss})]$. Finally, because in steady state $\phi[f(\omega_{ss})] = 0$, the long-run output/capital ratio is obtained as the solution to equation (27) above, which also follows by setting $g_e = 0$ in (30). Importantly, the presence of induced innovation generates a negative feedback from the labor share to itself in equation (29): such feedback changes the dynamics of the growth cycle, turning the Goodwin steady state from a center into a stable spiral.

There are three issues with the theory of induced technical change presented above: a first problem is that its logic involves a high level of abstraction, and it is not clear how to imagine an empirical counterpart to a strictly concave invention possibility frontier. Dumenil and Levy (1995; 2010) have used a stochastic setting that better conforms with intuition, and delivers very similar implications for the choice of factor-augmenting technologies. A second, more important criticism to the induced innovation hypothesis is that the relationship between income distribution and factor productivities arises from the choice of a point along an exogenous trade-off represented by the IPF. The position of the IPF is fully exogenous: the vertical intercept of the technical progress function in Figure (1) –which determines the long run growth rate of labor productivity– is a given of the theory. As such, the theory only explains the *direction* of technical change, that is how changes in income distribution determine variations in labor productivity growth, as opposed to capital productivity growth. But the theory is silent on the economic forces that give rise to a certain long-run growth rate, that is the innovation *intensity.* In other words, labor-augmenting technical progress is basically available without costs to the economy, and the determination of income distribution is, in fact, driven by an exogenously given trade-off between relative technological improvements.

5 Post-Keynesian Closures

The rejection of Say's law is a cornerstone of Keynesian economics, and implies that output is demand-determined rather than constrained by supply factors. Post-Keynesian economists have incorporated Keynes' ideas into class-based models in order to emphasize the distributive implications of demand-driven growth. Here, we distinguish between a Kaleckian closure, where the rate of capacity utilization adjusts to ensure the equilibrium between the supply of savings and the demand for investment, and a Kaldorian closure, where the equilibrium in the goods market occurs through changes in income distribution. This sharp distinction is helpful from an expositional point of view, even though some contributions feature both adjustment mechanisms simultaneously (Palley, 1996; Taylor et al., 2016; Sasaki, 2010).

5.1 Kaleckian Closure

As seen above, a distinctive element of Kaleckian economics is the dependence of investment demand on income distribution. While the classical tradition only considers the supply of savings and therefore sees economic growth as ultimately profit-driven, the Kaleckian investment function opens up the possibility of a paradox of costs to arise, so that aggregate demand and growth can be *wage-led*, in the now standard terminology.

We will analyze the role of investment demand and its relation with income distribution in the context of endogenous technical change. The original Kaleckian model (Kalecki, 1971) adds effective demand to a classical-Marxian framework. If firms charge a constant mark-up μ over real unit labor costs, the labor share is anchored to the value of the mark-up and fully determined by the equation

$$\omega = \frac{1}{1+\mu} = \bar{\omega},$$

which is formally equivalent to the classical closure (19). On the other hand, the short-run equilibrium between savings and investment is achieved through instantaneous adjustments in the utilization rate, which is a measure of aggregate demand. Using equations (15) and (17), and assuming in standard fashion that savings are more responsive than investment to changes in utilization, we can solve for the short-run equilibrium rate of capacity utilization and growth rate:

$$\begin{aligned} u^*(\bar{\omega},\gamma,s) = & \quad \frac{\gamma+\eta_1(1-\bar{\omega})}{sB(1-\bar{\omega})-\eta_0}; \\ g^*(\bar{\omega},\gamma,s) = & \quad sB(1-\bar{\omega})u^*(\bar{\omega},\gamma,s). \end{aligned} \tag{31}$$

Because $u^*_\omega \equiv \partial u^*/\partial\bar{\omega} > 0$, the paradox of costs holds: an increase in the labor share pushes up consumption while it depresses investment. However, the former effect offsets the latter, and aggregate demand is wage-led.[3] Further, both utilization and growth respond positively to autonomous investment: more bullish sentiments by investors result in a higher level of economic activity and in a higher growth rate in the short run. This is a version of the Keynesian metaphor of the 'widow's cruse'. Finally, the paradox of thrift holds: an increase in the saving rate lowers the level of economic activity.

In this model, growth can either be wage-led or profit-led. The simplest way to showcase the possibility of profit-led growth is to consider that, if $\eta_0 = 0$, that is if there is no accelerator effect and investment only responds to profitability, then $g^*_\omega = -\eta_1 < 0$, so that growth is profit-led. Through a continuity argument, it can be shown that profit-led growth will prevail when the accelerator effect on investment demand is small (or negative, which would mean that there are self-stabilizing forces at work on aggregate demand, as opposed to self-reinforcing mechanisms), but the sensitivity of investment to profitability is strong. On the other hand, autonomous investment always has a positive effect on growth. Finally, the paradox of thrift holds for economic growth, too. In fact, differentiating with respect to the saving rate we find:

$$g_s = \left[(1-\bar{\omega})Bu^* \left(\frac{s-1}{s}\right)\right] < 0,$$

because $s \in (0,1)$.

Let us now analyze the long-run interaction between distribution, demand, and technological change, using the three specifications of labor productivity growth given by (12), (13), and (14) in turn. With endogenous labor supply and the Kaldor law (12), $g^p = g_A + g_L = \varphi_0 + \varphi_1[g(\bar{\omega},\gamma,s) - g_L] + g_L$. In the short-run, labor productivity growth is endogenous and reacts positively to demand shocks. Moreover, it maintains the wage-led or profit-led character of the short-run accumulation rate. When the economy approaches balanced growth in the long run, the endogenous labor supply and productivity growth both accommodate the accumulation of capital stock. In steady state, the equality $g^p = g(\bar{\omega},\gamma,s)$ yields $g_A = \frac{\varphi_0}{1-\varphi_1}$ and $g_L = g(\bar{\omega},\gamma,s) - \frac{\varphi_0}{1-\varphi_1}$. While the accumulation rate is endogenous, labor productivity growth is semi-endogenous: the unlimited labor supply is the key adjusting variable that enables capital accumulation to progress unconstrained by supply factors.

If, on the other hand, labor supply is exogenous, we are back to the semi-endogenous growth case:

$$g(\omega,\gamma,s) = n + \frac{\varphi_0}{1-\varphi_1}. \tag{32}$$

Such a case is surprisingly similar to the classical model with exogenous labor supply. Satisfying (32), however, requires to endogenize either of the arguments determining the accumulation rate

[3]Post-Keynesian authors (the literature is vast: here, it is enough to mention Bhaduri and Marglin, 1990; Taylor, 2004, Chapter 8) have considered the possibility of aggregate demand being profit-led, so that $\partial u^*/\partial\omega < 0$. Especially with endogenous markups, this gives rise to a number of possible configurations of the interactions between aggregate demand and distribution. The focus of our paper is on the growth rate rather than the level of economic activity: the investment function we use is enough to generate profit-led growth.

on the left hand side of the equation. For instance, we can take up an insight from Skott (2010) and postulate that autonomous investment γ is the adjusting variable. Entrepreneurs would reduce accumulation when the employment rate rises, which occurs when growth is higher than its natural rate. Alternatively, in line with the neo-Kaleckian contributions beginning with Bhaduri and Marglin (1990), we can remove the assumption of an exogenous mark-up so that the labor share becomes the accommodating variable while autonomous investment remains exogenous. Regarding the comparative statics, total differentiation of (32) shows $d\omega_{ss}/d\gamma = -g_\gamma/g_\omega$. Accordingly, an increase in autonomous investment will raise (lower) the wage share when growth is profit (wage)-led. Finally, we can follow Ryoo (2016) who allows for adjustments in the saving rate to balance the labor market. Here, $ds/d\gamma = -g_\gamma/g_s$: given the paradox of saving an increase in investment pushes up the saving rate to keep the growth rate at its natural level.[4]

Let us now superimpose the classical-Marxian technical progress function (13) on the Kaleckian model. Being distribution exogenous, labor productivity growth is of the semi-endogenous variety, and does not add much to the analysis: $g_A = f(\bar{\omega})$. With unlimited labor supply, the growth rate of labor demand will again be the variable that adjusts in order to ensure balanced growth: it solves $g_L = g(\bar{\omega}, \gamma, s) - f(\bar{\omega})$. Such an adjustment process in labor supply emphasizes capital accumulation as the main driver of growth similarly to the classical framework. However, contrary to the classical case, a higher saving rate reduces growth: the paradox of saving holds exactly like in the short-run. Finally, autonomous investment is unconstrained in driving accumulation.

Conversely, if labor supply is exogenous, it is interesting to look at the case in which the markup and therefore the wage share is the endogenous variable. In this case, both accumulation and labor productivity growth are fully endogenous given that they increase in the labor share. Here, a shift in autonomous investment surely increases the share of labor when short-run growth is profit-led, since $d\omega/d\gamma = g_\gamma/(f_\omega - g_\omega)$. In the case of wage-led growth, instead, animal spirits and the wage share go together if the response of labor productivity growth to income distribution is stronger than the response of short-run accumulation, while they go in opposite directions if the converse is true. Exactly the opposite holds with regards to the effect of an increase in the saving rate on the wage share, given the paradox of thrift: $d\omega/ds = g_s/(f_\omega - g_\omega)$, which is negative if growth is profit-led, and positive if $g_\omega > f_\omega$, that is if accumulation is strongly wage-led. Thus, we see here a counterexample to the general case analyzed in this survey, according to which savings and the share of labor always move together. Since labor productivity is endogenous, under profit-led growth the paradox of saving holds in the long run as $dg_A/ds = f_\omega g_s/(f_\omega - g_\omega) < 0$; if on the other hand the economy is strongly wage-led, long-run productivity growth rises with the saving rate.

As mentioned before, the dependence of labor productivity growth on employment as in (14) provides a solution to the second Harrodian problem, with a Kaleckian closure and exogenous labor

[4]Skott (2010), Bhaduri and Marglin (1990), and Ryoo (2016) do not consider technical change in their analyses, but the logic of these contributions is compatible with the various determinations of labor productivity growth in our model.

supply (Palley, 2012). The employment rate, in this case, adjusts to ensure that

$$e_{ss} = h^{-1}[sB(1-\bar{\omega})u^*(\bar{\omega},\gamma,s) - n]. \tag{33}$$

Since $h(e)$ is increasing, the steady state employment rate retains the wage-led or profit-led character of the equilibrium growth rate, as well as the direct relationship with autonomous investment. Thus, even though utilization is always wage-led under the investment function (17), employment can be either wage-led or profit-led. Further, the 'widow's cruse' argument also applies to the employment rate, as $\partial e_{ss}/\partial\gamma > 0$. Finally, the paradox of thrift holds too, since $\partial e_{ss}/\partial s = h^{-1'}(g)\partial g/\partial s < 0$. On this regard, notice the stark difference with the classical models, where employment and saving go hand in hand. Long run productivity growth is fully endogenous as it rises with the employment rate. Similar analyses can be found in Bhaduri (2066); Flaschel and Skott (2006); Lavoie (2006); Sasaki (2010).

Related yet quite different is the solution proposed by Dutt (2006). He postulates the specific functional form $g_A = h(e) = e^{\theta}$, which implies $\dot{g_A}/g_A = \theta(g_L - n)$, where $\theta > 0$ is the elasticity of productivity growth to employment. By construction, labor productivity growth at a steady state of the model reconciles actual and potential growth since $g_L - n = g - g_A - n = 0$. Interestingly enough, Dutt (2006) takes the argument one step further, and imposes slow adjustments in autonomous investment in response to the same forces (though working in the opposite direction). This generates indeterminacy and path dependence in the model. For instance, if a tighter labor market discourages investment growth, we can assume $\dot{\gamma}/\gamma = -\psi(g_L - n), \psi > 0$. The two equations forming the corresponding dynamical system are linearly dependent on each other: the $\dot{\gamma} = 0$ and the $\dot{g_A} = 0$ nullclines coincide, and every point is an equilibrium point. Accordingly, the equilibrium productivity and output growth selection depends on initial conditions and history. An extension of the model (Dutt, 2010) posits that changes in both labor productivity growth and autonomous investment respond positively to the difference between the employment rate (as opposed to its growth rate) and its 'natural' level: $\dot{g}_A/g_A = \xi(e-\bar{e}), \xi' > 0, \xi(0) = 0; \dot{\gamma}/\gamma = \psi(e-\bar{e}), \psi' < 0, \psi(0) = 0$. Contrary to the previous case, the steady-state level of employment is fixed to some exogenous equilibrium level, but indeterminacy still affects the growth rate of output and productivity growth thanks to the long-run endogenous rate of capacity utilization.

5.2 Kaldorian Closure

As is well known, Kaldor (1972) explicitly rejected the notion of equilibrium as a relevant characterization of economic processes. In line with the Smithian tradition, but with a stronger emphasis on the demand side of the economy, he argued that the growth process is characterized by dynamic increasing returns, path dependence and cumulative causation between the size of the economy and technological progress. Macroeconomic theory, therefore, should be history-specific and hardly representable by equilibrium analysis. Still, beginning with Dixon and Thirlwall (1975), modern Kaldorian growth theory has produced equilibrium frameworks that convey analytically Kaldor's main insights on growth. It has been developed along three different, although intertwined, research agendas: balanced-of-payments-contrained growth, North-South growth models, and export-led cumulative causation growth. here, we focus on the last one as it features the most prominent role for technical change. Discussions and comparative surveys of the three approaches can be found in McCombie and Thirlwall (1994), King (2010) and Blecker (2013).

A standard representation of Kaldorian cumulative causation would follow Cornwall and Setterfield (2002) in assuming a demand regime, such as the one captured by the investment function (18), and a productivity regime given by the Kaldor-Verdoorn law (see also Naastepad, 2006). We model productivity growth according to the Kaldor-Verdoorn law in absolute terms, that is (11). Equating (18) and (11) yields

$$g^* = \frac{\gamma + \lambda\varphi_0}{1 - \lambda\varphi_1}; \quad g_A = \frac{\gamma\varphi_1 + \varphi_0}{1 - \lambda\varphi_1}, \tag{34}$$

whereas with endogenous labor supply employment growth can be found residually as $g_L = g^* - g_A = \frac{\gamma(1-\varphi_1)+\varphi_0(1-\lambda)}{(1-\lambda\varphi_1)}$. Both equilibrium capital and labor productivity growth are increasing functions of autonomous investment. Borrowing an expression from Taylor et al. (2016), 'demand drives growth all the way' and creates the necessary supply conditions. As noticed by Cornwall (1972), a 'Say's law in reverse' is at work in the Kaldorian process of growth: it is aggregate demand that creates its own supply, and not vice versa.

Kaldor (1956, p.94) suggested that the Keynesian principle of the multiplier can be alternatively used to provide a theory of the level of economic activity or of income distribution. Using (15) and (34) we have $suB(1-\omega) = \frac{\gamma+\lambda\varphi_0}{1-\lambda\varphi_1}$, which shows that in the short run investment generates the necessary savings either through changes in utilization or the wage share. We already explored the Keynesian framework with adjustments in capacity utilization in the previous section: here, we fix $u = 1$ and let income distribution be the accommodating variable. The equilibrium wage share can be found as

$$\omega = 1 - \frac{\gamma + \lambda\varphi_0}{sB(1 - \lambda\varphi_1)}. \tag{35}$$

The labor share moves inversely with autonomous investment, as the additional savings necessary to accommodate higher growth requires income distribution to shift in favor of the class with the higher propensity to save. Conversely, the saving rate has an opposite effect on distribution: since higher savings reduce demand, the additional spending necessary to keep the macroeconomic balance is obtained thanks to higher wages. Finally, improvements in technology or trade conditions $(\varphi_0, \varphi_1, \lambda)$ that increase equilibrium growth lower the wage share because, similarly to autonomous investment, they contribute to higher accumulation.

The picture changes dramatically if we assume exogenous labor supply growth. The balanced growth condition becomes

$$\frac{\gamma + \lambda\varphi_0}{1 - \lambda\varphi_1} = n + \frac{\gamma\varphi_1 + \varphi_0}{1 - \lambda\varphi_1}. \tag{36}$$

There is only one value of autonomous investment γ compatible with balanced growth, and it is fully determined by the innovation technology and the structural conditions affecting foreign trade. As a consequence, aggregate demand plays no role. Still, we are not quite be back to the semi-endogenous case: if a country's propensity to import and, in turn, the foreign trade multiplier depends on trade policy, the policy maker can affect the value of λ. In this case, growth becomes fully endogenous. Notice that if we represent a closed economy by assuming $\lambda = 0$, the steady state in (36) becomes equivalent to the semi-endogenous case discussed in classical closure with exogenous labor supply (save for the different specification of the Kaldor-Verdoorn law). This explains why some authors working within the Kaldorian tradition (Ryoo, 2016; Skott, 1989, 2010) assume that growth in mature economies is constrained by the exogenous rate of population growth.

The irrelevance of aggregate demand in the long run clearly does not sit well with Kaldor's view of the growth process. One possibility to re-establish the demand-driven nature of Kaldorian growth in the long-run is to assume that the structural parameters of the model react to disequilibrium in the labor market. Following Setterfield (2006; 2013a; 2013b), we can posit $\varphi_1 = \varphi_1(e)$,$\varphi_1^{'} > 0$. If a positive demand shock pushes growth above its natural rate ($g^* > g^p$), the employment rate rises because, given $g_L = g^* - g_A$, employment grows faster than labor supply. If the slope of the technical progress function increases as a response, the natural growth rate g^p rises and the initial demand shock has permanent growth effects. Palley (1996) has proposed a similar adjustment mechanism. He assumed that the innovation technology reacts to changes in capacity utilization rather than in the employment rate: with the equilibrium utilization rate being endogenous, this contribution combines the Kaleckian and the Kaldorian traditions.

The model is also capable of producing path dependence by imposing restrictions on its structural parameters. If we assume $\lambda\varphi_1 = 1$, the equilibrium growth rate is undefined as the demand schedule and productivity regime schedule have the same slope. If, in addition, we also assume that the two curves coincide we find a continuum of equilibria, whose selection will be determined by initial conditions and will be sensitive to demand shocks: growth is fully path-dependent (Palley, 2003; Setterfield, 2013b).[5] This conclusion, however, relies on the knife-edge condition.

The Kaldor-Verdoorn relation is a core building block of Kaldorian growth. Nevertheless, it is interesting to explore the implications of alternative forms of technical change in a post-Keynesian environment characterized by full capacity utilization and endogenous income distribution. Assume first that labor productivity growth follows equation (13). In the short-run, or in balanced growth with endogenous labor supply, the equilibrium is given by $sB(1-\omega) = \gamma + \lambda f(\omega)$, which implicitly solves for the equilibrium wage share $\omega^* = \omega(s, \gamma)$. Just as in (35), the wage share is inversely related to autonomous investment while it increases with the saving rate. The short-run equilibrium growth rate is $g^* = \gamma + \lambda f[\omega(s, \gamma)]$, which highlights the ambiguous effect of autonomous investment: on the one hand, autonomous demand directly increases the growth rate; on the other, however, its adverse distributional effect lowers the growth rate of labor productivity. Moreover,

[5]This scenario is not exactly consistent with our simplified framework since it would require $\gamma = -\varphi_0/\varphi_1 < 0$. In the general version of the model, however, the intercept of the investment function can be negative and still remain economically meaningful.

there is no paradox of saving: through its positive effect on the wage share, a higher saving rate increases labor productivity and output growth. Moving to the long-run, balanced growth with exogenous labor supply requires

$$\gamma = n + (1 - \lambda)f[\omega_{ss}(s, \gamma)].$$

Once again, demand is irrelevant in the long-run because autonomous investment must adjust to satisfy supply constraints. Growth is supply-driven, but the model is of the fully endogenous variety as the saving rate affects the equilibrium level of investment, of income shares, and labor productivity growth. A higher saving rate determines an increase in investment as $d\gamma/ds = (1 - \lambda)f[\omega_{ss}(s, \gamma)]/[1 - (1 - \lambda)f_\omega \omega_\gamma] > 0$, but the effect on income distribution and growth is in principle ambiguous, as $d\omega_{ss}/ds = \partial\omega_{ss}/\partial s + (\partial\omega_{ss}/\partial\gamma)d\gamma/ds \gtrless 0$.

Finally, if we let labor productivity growth rise with the employment rate as assumed in (14), the short-run equilibrium is $sB(1-\omega) = \gamma + \lambda h(e)$. With full capacity utilization, the employment rate becomes a state variable in the model. Labor productivity growth is given in the short-run and it does not affect the usual relation between income distribution, investment and the saving rate: growth is demand-led, as investment creates the required savings through changes in income distribution. More interesting is the long-run equilibrium of the model: the balanced growth condition is

$$\gamma = n + (1 - \lambda)f(e_{ss}).$$

Under this specification of technical change, demand matters even in the long-run. Autonomous investment increases capital accumulation, which determines a rise in the employment rate and, in turn, a higher natural growth rate. We are back to the 'widow's cruse' result found in (33), with both steady state employment and labor productivity growth increasing in response to stronger business confidence. The adjustment, however, occurs through a reduction in the wage share rather than an increase in capacity utilization.

6 Unbalanced Growth: Marx-Biased Technical Change

Despite the balanced growth requirement that $g_B = 0$, actual historical patterns have shown that the output-capital ratio may decrease for long periods of time. As an empirical illustration, Figure 2 plots Penn World Table series for the labor share, capital productivity, and the profit rate in United States, China, United Kingdom, and Japan over country-specific extended periods characterized by falling capital productivity (with trend lines for the US and the UK). The three variables under consideration are related through the basic long-run distributive curve (10), evaluated at full utilization:

$$r = B(1 - \omega). \tag{37}$$

With a roughly trendless labor share, decreasing capital productivity translates into a falling rate

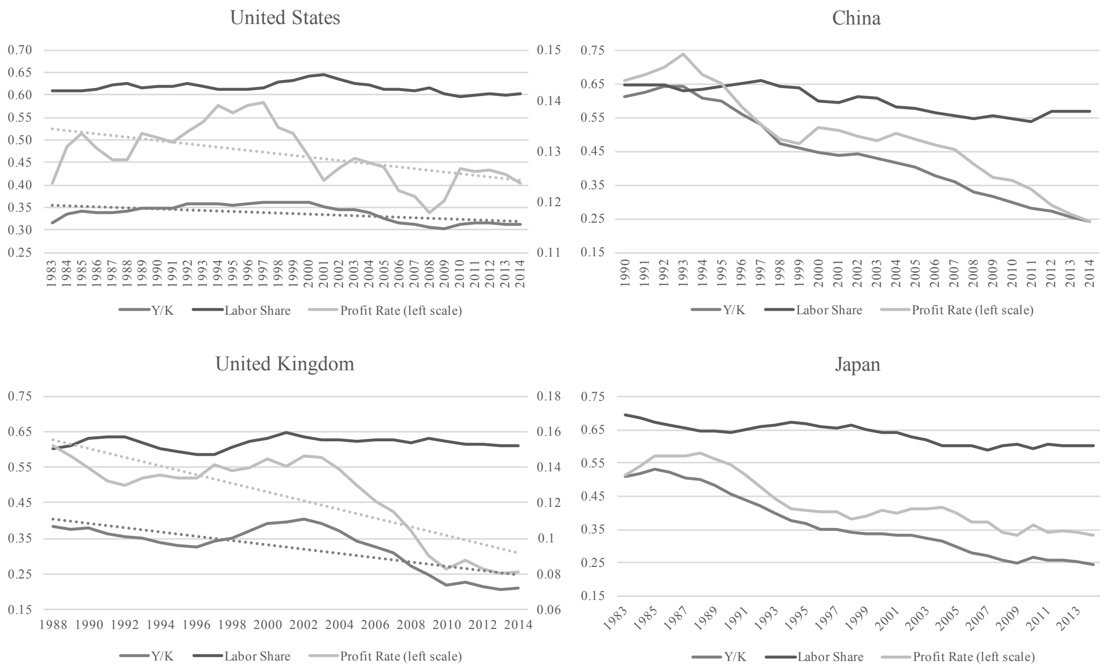

Figure 2: Marx-biased technical change for selected countries. Source: Penn World Tables 9.0 (Feenstra et al., 2015).

of profit of Marxian flavor. An almost constant labor share requires wages to grow in line with labor productivity: thus, the patterns displayed in Figure 2 are characterized by technical progress that is at the same time labor-augmenting ($g_A > 0$) and capital-using ($g_B < 0$) or, in the terminology by Foley and Michl (1999, Chapter 7), Marx-biased technical change (MBTC). The analysis by Dumenil and Levy (2010) also refers to this pattern of technical change.

One question that arises when staring at prolonged periods of falling profitability is why would firms adopt new production techniques that lower the profit rate relative to the existing techniques. In fact, a basic criterion for the choice of technique (Okishio, 1961) is to switch to a new technique if it does not decrease the profit rate at the current real wage. Such a criterion is very similar to the myopic quest for reduction in unit costs that drives the induced innovation hypothesis. For simplicity, consider discrete time and two techniques characterized by productivity parameters (A, B) and $((1 + g_A)A, (1 + g_B)B)$ respectively. For a given real wage, the prospective profit rate from the adoption of the new technique is

$$\begin{aligned} r' &= B(1 + g_B)\left(1 - \frac{w}{A(1+g_A)}\right) \\ &= B(1 + g_B)\left(\frac{1-\omega+g_A}{1+g_A}\right). \end{aligned}$$

For the technique to be adopted, the prospective profit rate must be no less than the current profit rate. The required inequality can therefore be solved in order to identify a threshold value for the profit share $1 - \tilde{\omega}$ such that, if the actual share of profits does not exceed this value, the technique will be adopted. Such *viability* criterion is:

$$1-\omega \leq 1-\tilde{\omega} \equiv \frac{g_A(1+g_B)}{g_A-g_B}. \tag{38}$$

Under MBTC, $1-\tilde{\omega}$ is always positive: thus, new techniques that increase labor productivity at the expenses of the output/capital ratio are viable. What firms do not foresee when switching to the new technique, however, is that maintaining a constant labor share, as time goes by, requires real wages to increase with labor productivity. As this process unfolds, the profit rate falls following the decline in the output/capital ratio. Foley and Michl (1999, Chapter 7) thus explain MBTC as analogous to a coordination failure: individually rational decision-making by firms results in a collectively self-defeating aggregate outcome. The historical patterns highlighted in Figure 2 leave little doubt about the relevance of MBTC. At the same time, an open modeling question is whether the coordination failure is a byproduct of the myopic nature of the firm's choice problem, and whether its extent could be reduced if firms adopted a longer horizon in their decision-making. Notice also that MBTC is compatible with transitional dynamics occurring in the NW quadrant of Figure 1.

An important implication of a Marx-biased pattern of technical change is that subsequent labor-augmenting innovations, together with the accumulation of capital stock that lowers the productivity of capital, produce capital deepening that is 'observationally equivalent' to the smooth process of capital-labor substitution in the Solow model. However, this process is driven by technical change, and not by substitution along a well-behaved production function, as pointed out by Michl (1999): capital deepening leaves a trace that appears like a production function, but it is in fact just the *fossil* record of past technology. The appeal of such pattern is therefore obvious for alternative economists, because MBTC is immune from the logical flaws of the neoclassical production function that emerged from the Cambridge capital controversy of the 1960s.

Although beyond the scope of this paper, the viability criterion provided by (38) has also been used to test the empirical relevance of competing theories of income distribution, namely the classical as opposed to the neoclassical theory. Michl (1987) devised an early test that appears to be strongly rejecting the neoclassical theory for OECD countries. Basu (2010) has extended the logic of the test to a stochastic setting through the use of cross-country regressions: his finding confirm the rejection of the marginal theory of distribution.

7 Costly Innovation

Until recently, researchers working within alternative frameworks have not considered the costly nature of the innovation process. Even though, to the best of our knowledge, there is no critical literature to draw from, one can think of a several explanations for this choice. First, in post-Keynesian economics, output and growth are demand-determined, supply only needs to accommodate demand conditions, and the assumption of a Kaldor-Verdoorn law such as (22) already well captures the response of labor productivity growth to investment demand. Second, no matter whether one takes a classical or post-Keynesian viewpoint, inventive activity is highly uncertain, and does not lend itself to easy formalization as the output of a production process. Third, considering the costs incurred by firms when innovating requires to dive into the microeconomic allocation problem of choosing how to split resources between capital accumulation and inventive activity, which typically involves a profit-maximization problem. This may generate some skepticism if profit-maximation is identified with neoclassical marginal cost pricing.

And yet, in a world without a well-behaved production function but instead with labor and capital as perfect complements, investing in labor-augmenting innovation confers firms the ability to save on unit labor costs, and thus has the potential to increase profits. Since there is no question that all new technologies used for private production involve at least some expenditure, it is precisely the conflictual nature of the innovation process that calls for the inclusion of costly R&D into alternative models of growth and distribution. These considerations are at the heart of recent contributions by Tavani and Zamparelli (2015) and Zamparelli (2015), which introduce the problem of resource allocation on costly R&D by capitalist firms in the classical model with exogenous labor supply. We can follow the neoclassical endogenous growth theory in assuming that the flow of new ideas $\dot{A}$ depends positively on R&D inputs on the one hand, and linearly on the existing level of technology itself on the other. As pointed out above, a linear spillover from past ideas is useful to generate sustained growth in the long run. Contrary to (3), however, it is forgone consumption, rather than labor, to be employed in the production process of new technology. Accordingly,

$$\dot{A} = (R/Y)^{\zeta} A, \tag{39}$$

where R is the amount of resources invested in R&D, homogeneous with output and capital stock, $\zeta \in (0, 1)$ is the constant elasticity of innovation to R&D per unit of investment, and the normalization of R&D spending is necessary to avoid explosive growth.

Once inventive activity becomes costly, capitalists have two alternative uses available for their saved profits: capital accumulation and innovation. Both types of investment raise total profits, but in different ways: innovation reduces unit labor costs in production, while capital accumulation increases the size of a firm's business. If we let δ be the share of saved profits invested in R&D, the growth rate of labor productivity is:

$$g_A = \dot{A}/A = [s\delta(1 - \omega)]^{\zeta},$$

while physical capital accumulation obeys:

$$g_K = \dot{K}/K = s(1 - \delta)B(1 - \omega).$$

The representative capitalist will choose δ to maximize some measure of profitability. The solution to the problem will be an allocation of investment as a (possibly implicit) function of all variables involved: saving preferences, technology and unit labor cost $\delta^* = \delta(s, B, \zeta, \omega)$. Then, using the profit-maximizing allocation in the balanced growth condition we can write:

$$s[1 - \delta(s, B, \zeta, \omega_{ss})]B(1 - \omega_{ss}) = [s\delta(s, B, \zeta, \omega_{ss})(1 - \omega_{ss})]^{\zeta} + n. \tag{40}$$

In line with the standard results of the classical model with limited labor supply, equation (40) shows that steady state income distribution is endogenous, although the effect of a change in the saving rate on the labor share cannot be established without further information on δ^*. In equilibrium, labor productivity growth depends on the saving rate both directly and indirectly through its effect on investment allocation and income distribution: $g_A^* = [s\delta(s, \omega_{ss})(1 - \omega_{ss})]^{\zeta}$. Tavani and Zamparelli (2015) obtain the values of investment in both capital stock and R&D as the solution of the intertemporal utility maximization of forward-looking capitalist households with perfect foresight. The relation between saving preferences, income distribution and growth is in principle ambiguous, but a numerical implementation of the model calibrated to match long-run US data shows that a higher saving rate increases labor productivity growth as well as the labor share.

Bridging the induced innovation hypothesis with endogenous growth considerations, Zamparelli (2015) assumes that the position of the innovation possibility frontier depends on R&D expenditure, so that firms choose capital accumulation and the intensity and direction of technical change by solving a myopic profit maximization problem. In steady state, increases in the saving rate raise long run per capita growth, and have a positive effect on the labor share.

In Section 4, we discussed the two possible outcomes of the classical growth framework: endogenous growth with exogenous distribution, and endogenous distribution with exogenous (or semi-endogenous) growth. On the other hand, Section 4.3 has shown how the induced innovation hypothesis implies that both growth and income distribution are basically exogenous and determined solely by Kennedy's innovation possibility frontier. The introduction of costly innovation has therefore substantial implications for the classical model, in that it delivers the simultaneous endogeneity of growth and income distribution, with or without the induced innovation hypothesis. Moreover, in contrast with Romer (1990) contribution, this model delivers a fully endogenous growth rate and labor share even though it is foregone consumption, and not output, that provides resources to the R&D process. The main element that allows for this result is that both the total amount of saving and its allocation between capital accumulation and R&D investment depend on income distribution, unlike in the Romer model.

8 Conclusion

In this paper, we surveyed a series of recent contributions on endogenous technical change made by economists working in alternative traditions. We identified three main views on the determinants of technical change one can find in the literature: (i) a classical-Marxian hypothesis that links factors productivity growth to income distribution, (ii) a Kaldorian hypothesis that sees labor productivity growth as a byproduct of capital accumulation, and (iii) a hypothesis made in recent post-Keynesian literature according to which the growth rate of labor productivity is related to labor market tightness as measured by the employment rate. These alternative viewpoints can be embedded into growth models classified by their different closures. We first investigated a supply-side classical-Marxian closure based on Say's law, and we distinguished between a labor-abundant and a labor-constrained version of the model. Second, we explored two Keynesian, demand-side closures with independent investment functions: a Kaleckian closure in which quantity adjustments in the rate of capacity utilization achieve the equilibrium between savings and investment; and a Kaldorian closure where the equilibrium in the goods market is brought about by changes in income distribution. We studied these Keynesian cases both with and without labor constraints.

One element of novelty in our account of these contributions is the comparison with neoclassical exogenous and endogenous growth theories based on product variety. A first point worth stressing is that the balanced growth paths of all the theories we surveyed match the Kaldor facts in the very long run. In addition to this, we find the comparison illuminating along several dimensions: we summarize the main implications of classical supply-side models and demand-driven models in turn, focusing on the various models' steady states.

First, we showed that there are strong similarities between a classical-Marxian model with endogenous labor supply (Foley and Michl, 1999, Chapter 6) and a version of the product-variety growth model where R&D is financed by foregone consumption, an example of which can be found in Aghion and Howitt (2010, Chapter 3). In both frameworks, the labor share is exogenously given, but the growth rate is endogenous and increases in the saving rate: it can potentially be affected by policy action. Second, we highlighted the analogy between a classical model with exogenous labor supply together with a Kaldorian technical progress function (Taylor, 2004, Chapter 5) and the semi-endogenous neoclassical growth model by Jones (1995). In both cases, the long-run rate of growth of labor productivity is determined endogenously but is independent of saving preferences and it is therefore policy-invariant. However, income distribution is fully endogenous, in that the labor share is an increasing function of the saving rate. The endogeneity of income distribution occurs even in the Solow model, which however leaves the explanation of labor productivity growth unaddressed. Third, the classical model with induced technical change resembles the neoclassical exogenous growth model as it features an adjusting long-run output-capital ratio, inversely related to the saving rate; however, contrary to the neoclassical model, income distribution is independent of saving preferences and fully determined by the shape of Kennedy's innovation possibility frontier. Fourth, the Romer (1990) model with labor as R&D input determines both growth and the labor share endogenously as increasing functions of the saving rate, similarly to classical models with costly R&D (Tavani and Zamparelli, 2015; Zamparelli, 2015). The latter contributions, however, achieve the endogeneity of factor shares even with forgone output as an input to the R&D process: key to this result is that both capital accumulation and R&D investment depend on income distribution. Fifth, even in the sole unbalanced growth case we described, there is a striking parallel to be drawn between the capital deepening arising from the capital-labor substitution that drives the transitional dynamics of the Solow model and a Marx-biased pattern of technical change.

Despite these remarkable similarities, the differences between classical and neoclassical models are profound. To begin with, none of the classical models presupposes full employment of labor, which is a crucial element of all their neoclassical counterparts. Second, the neoclassical models hinge on a representative household, while the classical models are class-based. Third, the classical models are immune from the Cambridge critique of the neoclassical production function. Fourth, the classical models are in principle compatible with growth-distribution cycles around the steady state, while the convergence to the steady state is monotonic in the Solow model and the Romer model has no transitional dynamics. Cycles in labor productivity growth and a measure of the real wage may occur in the Aghion and Howitt (1992) model, but the economy is always at full employment.

Demand-driven models in the post-Keynesian tradition add the rejection of Say's law to the features that put classical models in contrast with neoclassical growth. Since saving and investment decisions do not coincide, it is autonomous investment that provides the key behavioral variable to look at the relation between capital accumulation, technical change and distribution. First, in the short-run or if labor supply is endogenous, both Kaleckian and Kaldorian models feature a fully endogenous growth rate increasing in autonomous investment: the difference lays in what is the adjusting variable in the two models. Under the Kaldorian closure, income distribution is the accommodating variable similarly to the classical model with exogenous population growth or the semi-endogenous neoclassical model: the labor share rises with the saving rate. In Kaleckian models, it is the quantity adjustment in capacity utilization that ensures the saving-investment equilibrium; from this point of view there is a similarity with the classical model with exogenous distribution where, following an increase in the saving rate, higher employment growth accommodates the increase in the growth rate. Second, if labor supply is exogenous and productivity growth follows the Kaldor-Verdoorn law, Keynesian models lose their demand-led flavor in the long-run: the balanced growth condition requires autonomous investment to be the adjusting variable given population growth, innovation technology and (in the Kaldorian case) trade conditions. We thus explored multiple ways to achieve demand-determined endogenous growth with exogenous population: making labor productivity growth depend on the level of endogenous variables such as the labor share or the employment rate; assuming that innovation technology reacts to differences between actual and potential output; and generating path-dependence by assuming that capital accumulation and labor productivity growth are defined by the same equation.

Overall, our account has shown that changes in accumulation, be they supply- or demand-driven, tend to have distributional rather than growth effect when the labor force grows at an exogenous rate. The analytical accomplishment of endogenous growth theories, both neoclassical and alternative, has consisted in devising specifications of technological change that make the natural growth rate sensitive to investment decisions. They all rely on knife-edge conditions: as noticed by Solow (2007), endogenous growth requires a linear differential equation in the level of technology. However, a key difference is the emphasis on different factors which may be responsible for long-run growth. Neoclassical growth theorists have emphasized the role of intertemporal saving preferences and the market structure; alternative theorists have focused on income distribution, the state of the labor market, and investors' behavior as key determinants of the pace of technical progress.

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