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Public Goods, Redistribution, and Growth: A Classical Model

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Public Goods, Redistribution, and Growth: a Classical Model

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Preliminary Version

Abstract

We extend the basic Classical growth model by introducing a productive and redistributive role for the public sector in an economy populated by two classes, workers (who supply labor, consume, and do not save) and capitalists (who own capital stock, consume and save). The government levies a tax on profits in order to: (i) finance the provision of a public good that augments the production possibilities of the economy, and (ii) integrate labor incomes through a transfer to workers. Following Michl (2009), we focus on two different model 'closures', which deliver an endogenous and an exogenous growth rate respectively. In both cases, the analysis of taxation and government spending composition between public goods and transfers requires to specify the government's preferences. In the endogenous growth model, the government's choice fixes long-run growth and income distribution. In the exogenous growth model, policy decisions determine income distribution and the employment rate.

Keywords: Classical growth, functional distribution, redistributive policy

JEL Classification: D33, E11, O38

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1 Introduction

The analysis of the productive role of public capital accumulation and public services is firmly established in the mainstream economic literature. Following the seminal paper by Arrow and Kurz (1969), several contributions (see among others Fisher and Turnovsky, 1995; Bajo-Rubio, 2000; Carboni and Medda, 2011) analyzed how the size and composition of public capital and expenditure affect growth within the neoclassical exogenous growth framework. Provided that private capital and public expenditure are complements, and that public expenditure is financed through a tax on private agents, growth is a hump-shaped function of the tax rate. On the one hand, taxes contribute to growth because they finance public capital accumulation and/or public services provision; on the other hand, they have a negative effect on growth in that higher taxation reduces the incentives to private capital accumulation. The optimal tax rate is such that the two effects exactly offset each other, and it is found at the point where the marginal product of public expenditure equals the after tax marginal product of private capital. When growth is exogenous, public expenditure has permanent effects on the long-run level of output per capita, while it affects the growth rate only during the transition to the steady state.

At the onset of the endogenous growth literature, Barro (1990) produced an endogenous growth model with a public sector. Under constant returns to scale in private capital and expenditure in public services, the size of the government sector has permanent effects on the growth rate, which is maximized when the tax rate equals the elasticity of output with respect to government spending. Barro's analysis has been later extended in multiple directions to consider: different types of productive public expenditures (Devarajan et al, 1996; Agenor and Neanidis, 2011); the distinction between productive and non-productive spending (Glomm and Ravikumar, 1997; Kneller et al, 1997); and alternative ways of financing public expenditure (Corsetti and Roubini, 1996; Turnovsky, 1996).

The empirical literature has widely investigated the contribution of public expenditure to economic growth, with inconclusive results. Depending on econometric specifications and samples under consideration, different studies have found that marginal product of public capital may uniformly exceed (Aschauer, 1989; Kocherlakota and Yi, 1996) or be lower (Holtz-Eakin, 1994) than the marginal product of private capital; others (Aschauer, 2000) have found the expected non-linear relation between public capital and growth.

Summing up, the government's role of provider of productive public services and its ability to influence economic growth appear to have been thoroughly investigated. On the contrary, relatively little effort has been devoted in analyzing the joint productive and redistributive role of the government sector in a growing economy. In fact, the 'representative agent' assumption characterizing most contributions in the exogenous and endogenous growth literature, makes the focus on distributional issues less compelling, as both wage and interest incomes accrue to the same representative agent.

The starting point of this paper is that class matters. According to a well-established tradition in the non–mainstream literature (Kaldor, 1956; Pasinetti, 1974; Marglin, 1984), an explicit division of society between workers and owners of capital assets can shed light not only on an economy's growth path, but also on the interaction between income and wealth distribution on the one hand, and economic outcomes. While obviously a simplification, a sharp class division becomes a useful tool to analyze the actions of a policy-maker concerned with both growth and the distribution of income. Given these premises, in this paper we study the productive and the redistributive role of the public sector in a Classical-Marxian growth model. Following Michl (2009), we focus on the role of different 'closures' to the Classical model in order to draw policy implications.

First, we consider an economy with unlimited supply of labor. In particular, we look at the public sector in what Foley and Michl (1999) have called 'conventional wage share' model, featuring a perfectly elastic labor supply in correspondence of a given wage share, so that employment is determined by labor demand only. A fundamental feature of this model closure is that it delivers an endogenous growth rate in the economy. In such a framework, the government generates revenues by taxing profits, and uses the proceedings to: (i) finance the provision of a public good that directly affects the production possibility of the economy, and (ii) finance expenditures on transfers to workers, in order to improve their distributional position and consumption standards.

In order to determine the government's fiscal strategy, we introduce a payoff function for policy makers. They value both the growth rate of the economy and workers' after-tax income share which, in turn, determines their consumption standards. The rationale is to balance the need to increase the overall social wealth with the concern for redistribution towards the less privileged class in the economy. The relative weight on the two alternative policy goals pins down both the growth rate and the after tax factor distribution of income in the economy: the higher the weight on redistribution, the higher the highest potential social consumption and the lower the achievable growth rate. In this respect, our analysis delivers similar results to those found in Alesina and Rodrik (1994), although their contribution only applies to economies at full employment.

Second, we analyze the case of a labor-constrained economy, where the growth rate of real GDP per worker is equal to the exogenous growth rate of population. In this model, it is income distribution that adjusts to make capital accumulation compatible with the exogenous growth rate in order to maintain long run employment constant. Under this alternative closure, the government's redistribution in favor of the working class occurs by providing income subsidies to unemployed workers. Since growth is fixed at the natural growth rate, it cannot enter as an argument in the policy maker's payoff function: it turns out that, within this alternative model, maximizing the labor share of income requires to select a tax rate equal to the elasticity of GDP to public expenditure, and to transfer no income to the unemployed. As soon as the government starts weighing the welfare of unemployed workers, however, it will select a tax rate in excess of its labor share maximizing value.

With its focus on the simultaneous analysis of the productive and redistributive role of the public sector in a growing economy, our contribution fills a gap in the non–mainstream literature on growth and distribution. Authors working within the Classical-Marxian tradition have mostly introduced the government sector to analyze how alternative social security systems (Michl and Foley, 2004; Michl, 2007) on the one hand, and public debt and deficits (Michl, 2006) on the other, affect economic growth and the distribution of income and wealth. These contributions did not consider, however, the productive role of public investment. In a recent paper, Commendatore et al (2011) study the growth effects of different types of government spending in a post-Keynesian framework, and show that these effects depend on after-tax profits. Differently from our model, they do not consider an explicit redistributive role for the government and do not solve for the equilibrium composition of public expenditure.

The remainder of the paper is organized as follows. Section 2 describes the technology in use in the economy. Section 3 studies the endogenous growth model, both analytically and numerically, while Section 4 repeats the exercise for the exogenous growth model. Section 5 concludes.

2 Technology

Production of the final good Y requires fixed proportions of effective capital stock BK in combination with effective labor AL:

$$Y = \min\{BK, AL\}. \tag{1}$$

The parameter $A > 0$ is set to be constant in the analysis. Without loss of generality, we impose $A = 1$ throughout.[1] The output/capital ratio, instead, is not constant, but depends on a flow of public productive goods (or services), normalized by the size of capital stock $\chi \equiv G/K$, as follows: $B = \chi^{1-\gamma}$, $\gamma \in (0, 1)$. Individual firms take the level of χ as a given at any point in time. Because of the Leontief structure of the aggregate production function, profit–maximization by firms requires not to employ unproductive resources so that, at any point in time, $BK = L$.

The description of technology enables us to highlight the role of public spending in the trade–off between social consumption and growth. Following Foley and Michl (1999), we refer to such trade–off as the growth–distribution (GD) schedule. Output can be used for consumption (C), accumulation of private capital or public productive services. Accordingly, $Y = C + \dot{K} + G$. From (1), $Y/L = A = 1 = C/L + \dot{K}/L + G/L$. Notice also that $\dot{K}/L = \frac{\dot{K}}{L}\frac{K}{K} = g_K\frac{K}{L} = g_K/B = g_K\chi^{\gamma-1}$; on the other hand, $G/L = \frac{G}{L}\frac{K}{K} = \chi\frac{K}{L} = \chi\frac{A}{B} = \chi/B = \chi^{\gamma}$. By plugging these definitions in the output uses identity, and denoting aggregate consumption per worker by $c \equiv C/L$, we derive the social consumption–growth schedule as:

$$g_K = \chi^{1-\gamma} - \frac{C}{L}\chi^{1-\gamma} - \chi = \chi(\chi^{-\gamma} - 1) - c\chi^{1-\gamma}. \tag{2}$$

There is a continuum of GD schedules, each representing a technique of production associated to a specific χ. In fact, by choosing the amount of public good provided in the economy, the government selects the GD schedule which will be the technological constraint for output uses and the distribution of income.

3 Conventional Wage Share Model

Growth models with unlimited labor supply trace back to at least the analysis of a dual economy by Lewis (1954). We begin by assuming labor supply to be infinitely elastic at the exogenous wage rate (share). An exogenous share of labor in real output has the traditional interpretation of a subsistence wage (adjusted for labor productivity) in Malthusian frameworks, or the value of the productivity–adjusted wage that guarantees the reserve army of labor to remain constant over time in Marxian frameworks.

[1] Thus, the pre-tax labor share and the pre-tax real wage rate are equal throughout this paper.

The public sector finances its expenditures by levying a proportional tax $\tau \in [0,1]$ on profit incomes Π. Public revenues have two uses. On the one hand, they will be used to finance the provision of public goods in production. On the other hand, they will finance transfers to workers to integrate wage incomes. Let $\delta \in [0,1]$ be the transfer to wages as a share of public revenues. Denoting the labor share by ω, we have:

$$\chi = \tau(1-\delta)\Pi/K = \tau(1-\delta)(1-\omega)Y/K = \tau(1-\delta)(1-\omega)\chi^{1-\gamma}. \tag{3}$$

Hence, the equilibrium level of the public good, normalized by the size of capital stock, is

$$\chi = [(1-\delta)\tau(1-\omega)]^{\frac{1}{\gamma}}. \tag{4}$$

On the other hand, the after tax labor share $\tilde{\omega}$ is given by:

$$\tilde{\omega} = \omega + \tau\delta\Pi/L = \omega + (1-\omega)\tau\delta Y/L = \omega + \tau\delta(1-\omega). \tag{5}$$

As it is typical in models in the Classical tradition, we assume that workers do not own stakes in capital accumulation, and consume all of their (disposable) wage income at each period. Hence, the after tax labor share in (5) will also provide a measure of consumption by a typical worker in the economy. Owners of capital goods (capitalists), on the other hand, choose how much of their disposable income to allocate between private capital accumulation and consumption each period. We assume that they save a constant fraction s of their after tax profit income to accumulate capital stock. Accordingly, we find the growth rate of private capital stock as:

$$g_K \equiv \frac{\dot{K}}{K} = s(1-\tau)(1-\omega)Y/K = s(1-\tau)(1-\omega)\chi^{1-\gamma}. \tag{6}$$

Given (4), we can express the accumulation rate —in turn equal to the growth rate of output— as a function of the tax rate and the transfer to workers:

$$g(\tau,\delta) = s(1-\tau)\tau^{\frac{1-\gamma}{\gamma}}(1-\delta)^{\frac{1-\gamma}{\gamma}}(1-\omega)^{\frac{1}{\gamma}}. \tag{7}$$

Inspection of (7) shows that the growth rate is decreasing in the transfer. Since transfers only finance consumption, while subtracting from investment and the provision of the public good, they necessarily reduce the growth rate. Conversely, the growth rate is hump–shaped in the tax rate: on the one hand, an increase in taxes raises capital productivity; on the other hand, it lowers the net profit rate thus reducing funds available for private accumulation.

Growth is increasing (decreasing) in the tax rate as long as the first effect dominates (is dominated by) the second one, which occurs for relatively low (high) levels of τ. The growth–maximizing tax rate is readily found, independently of the transfer δ, as the elasticity of the output/capital ratio with respect to χ:

$$\tau^* = 1 - \gamma. \tag{8}$$

Accordingly, the highest achievable growth rate is

$$g_{max} = s\gamma(1-\gamma)^{(1-\gamma)/\gamma}(1-\omega)^{1/\gamma}.$$

On the other hand, social consumption per worker will be given by:

$$c(\tau, \delta) = (1-s)(1-\tau)(1-\omega)Y/L + \tilde{\omega} = \omega + ((1-s)(1-\tau) + \tau\delta)(1-\omega). \tag{9}$$

Consumption is increasing in the transfer due to the positive impact of the latter on the after tax wage share; whereas higher taxes will increase consumption only if the positive effect on transfer more than offset the negative effect on capitalists' consumption, which occurs if $\delta > 1 - s$.

3.1 Some efficiency considerations

Since τ^* maximizes the growth rate for any level of transfer share δ, one may be tempted to separate the problem of growth maximization from that of redistribution by charging the growth maximizing tax rate and using δ for redistribution. However, such a procedure would not yield an efficient result. In fact, it can be shown that, at high levels of consumption per worker, the growth maximizing tax rate exceeds $1-\gamma$. In order to illustrate this point, let us first obtain the set of feasible growth-consumption allocations as functions of the tax rate. Solve (9) for δ to find:

$$\delta = \frac{c - (1-s)(1-\tau)(1-\omega) - \omega}{\tau(1-\omega)}.$$

Then, plugging δ into (7) yields

$$g(\tau, c) = s(1-\omega)(1-\tau)\left[((1-s) + \tau s)(1-\omega) + \omega - c\right]^{\frac{1-\gamma}{\gamma}}, \tag{10}$$

which represents the downward sloping growth-consumption frontier for any tax rate. When $\tau = \tau^* = 1 - \gamma$, the frontier becomes

$$g(1-\gamma, c) = s(1-\omega)(\gamma)\left[((1-s) + (1-\gamma)s)(1-\omega) + \omega - c\right]^{\frac{1-\gamma}{\gamma}}.$$

Next, fix c at a threshold level $\bar{c}$ and look at the set of tax rates such that $g(\tau,\bar{c}) > g(1-\gamma,\bar{c})$. This inequality is solved for $c > \bar{c} = (1-s)(1-\omega)+\omega+s(1-\omega)\frac{(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}-\tau(1-\tau)^{\frac{\gamma}{1-\gamma}}}{\gamma^{\frac{\gamma}{1-\gamma}}-(1-\tau)^{\frac{\gamma}{1-\gamma}}}$. The interpretation is that, despite $\tau^* = 1-\gamma$ being the unconstrained growth maximizing tax rate, if desired social consumption is fixed at sufficiently high levels growth maximization requires a tax rate higher than τ^*. In other words, $g(\tau,c) > g(1-\gamma,c)$ for $c \in (\bar{c},1) \cap \tau \in (1-\gamma,1)$. 1 illustrates the point graphically.

Notice that the threshold value $\bar{c}$ is a function of the tax rate itself, and it is economically meaningful only when lower than the highest consumption achievable $c_{max} = 1$. We show in the Appendix that $\bar{c} < 1$ for $\tau > 1-\gamma$, which ensures that the set $c \in (\bar{c},1) \cap \tau \in (1-\gamma,1)$ is non empty.

The above discussion shows that, for fiscal policy to be set at efficient levels, the choice by the government over financing the public good and redistribution cannot be sequential, but must be taken at the same time. To this aim, we need to define the government's preferences over growth and redistribution.

[FIGURE 1ABOUT HERE]

3.2 Government Preferences and Policy

Suppose that the government weighs both growth and workers' consumption when determining the tax rate and the amount of transfers in the economy. On the one hand, a weight on growth reflects a concern for both the evolution of aggregate wealth and capitalists' after tax income, given by $\tilde{r} \equiv (1-\tau)\Pi/K = g/s$. On the other hand, when choosing a tax rate and an amount for the transfer, the government also considers workers' standards of living, as measured by the after tax labor share $\tilde{\omega}$ —in turn equal to workers' consumption. In what follows, we restrict the government's preferences between growth and workers' income share to be log–linear. Letting $\beta \in [0,1]$ denote the weight assigned to workers' consumption, the government's problem is:

$$\begin{aligned} \text{Choose } (\delta,\tau) \quad \text{to max} \quad & \beta \ln \tilde{\omega}(\delta,\tau) + (1-\beta)\ln g(\delta,\tau) \\ \text{s.t.} \quad & (\delta,\tau) \in [0,1]\times[0,1] \end{aligned} \tag{11}$$

Starting with the transfer, the first order necessary condition reads:

$$\frac{\beta}{\tilde{\omega}}\tau(1-\omega) = (1-\beta)\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1}{1-\delta}\right),$$

and solves for the following value of the transfer as a function of the tax rate and government's preferences

$$\delta(\tau;\beta) = \frac{\beta\gamma - \frac{(1-\beta)(1-\gamma)\omega}{\tau(1-\omega)}}{\beta\gamma + (1-\beta)(1-\gamma)}. \tag{12}$$

Equation (12) emphasizes the role of the government's preferences between growth and redistribution in determining the size of the transfer. If $\beta = 0$, the denominator of the fraction is positive but the numerator turns negative. Since negative transfers are not admissible, the chosen transfer will be zero. The intuition is that, when $\beta = 0$, only growth matters for the government, and the growth rate is maximized when there is no transfer to wage incomes. Conversely, for $\beta = 1$, only distribution matters, and the transfer will reach its maximum value $\delta = 1$.

The numerator of the fraction appearing in the RHS of (12) is also useful to identify a threshold for the distributional weight β that ensures a positive value of the transfer for a given tax rate. The threshold value is

$$\bar{\beta} = \frac{(1-\gamma)\omega}{(1-\gamma)\omega + \gamma\tau(1-\omega)}. \tag{13}$$

If $\beta > \bar{\beta}$, policy makers will choose a positive transfer to integrate workers' income. The threshold $\bar{\beta}$ is important in this framework, because as soon as the policy maker's weight on redistribution crosses that value, fiscal policy will act in affecting the technique in use in the economy relative to the one attainable at the mere growth-maximizing tax rate. It is particularly interesting to check how the threshold $\bar{\beta}$ responds to the conventional wage share.

$$\frac{\partial\bar{\beta}}{\partial\omega} = \frac{(1-\gamma)\gamma\tau}{[(1-\gamma)\omega + \gamma\tau(1-\omega)]^2} > 0. \tag{14}$$

A higher wage share lessens the pressure for redistribution in favor of workers. Therefore, a positive transfer will require a higher preference for redistribution by the policy maker.

Let us now turn to the choice of the tax rate. The first order necessary condition can be written as:

$$\frac{\beta}{\tilde{\omega}}(1-\omega)\delta = (1-\beta)\left(\frac{\tau+\gamma-1}{(1-\tau)\tau\gamma}\right) \tag{15}$$

and identifies an implicit function $\tau(\delta;\beta)$ relating the choice of the tax rate to the choice of the transfer, given government's preferences. It is easy to see that, when $\beta = 0$, the transfer does not matter, and the tax rate will be equal to the growth–maximizing value $\tau^* = 1 - \gamma$.

Given the strong nonlinearities characterizing the first–order conditions on the choice of the tax rate, this is as far as we can go with analytical speculations, and therefore we proceed illustrating the insights that can be drawn from the numerical analysis. For these simulations, we pick a(n illustrative) value for the elasticity parameter γ=.75, which returns a growth-maximizing tax rate of 25%, and a saving rate out of profits equal to .5. Then, we let the preference for redistribution parameter β vary from 0 to 1, at 10% intervals. For values of the redistribution weight β up to .5, the government's pay-off is strictly decreasing in the transfer. Hence, δ will be set equal to zero, and the tax rate will be set to its growth–maximizing value τ^*. Starting at $\beta = .6$, however, the government's objective functional becomes non–monotone in the transfer. Thus, both the chosen values for the tax rate and the transfer will be interior. This will be true up to the extreme case where $\beta = 1$, corresponding to which the objective function becomes uniformly increasing in both the tax and the transfer. In this scenario, the chosen tax rate and transfer will be the highest possible in the admissible domain: $\tau = \delta = 1$. Table 1 reports the chosen values of taxes and government spending composition corresponding to increasing values of β, up to .9.

[TABLE 1 ABOUT HERE]

3.3 The Growth-Distribution Schedule

As the government's preferences over growth and distribution change, different choices of taxes and compositions of government spending between the provision of public goods and transfers to workers will be implemented. It is important to stress that, given (4), one effect of such choice will be that of selecting the technique of production in use in the economy and the corresponding GD schedule. Moreover, given the chosen values of τ and δ, both the growth rate of the economy as found in equation (7) and the value of social consumption per worker as in equation (9) will be fully determined.

In Figure 2 below, we plot the different consumption–growth schedules that arise when β is allowed to vary between 0 and 1, as well as the corresponding growth and consumption points. For any value of $\beta \in [0, .5]$, the

GD schedule does not move from the dark orange line in the figure, and the actual growth and distribution point is fixed at A. This is our benchmark technique, which the government implement by setting $\delta = 0$ and $\tau = 1 - \gamma$. However, as soon as the redistribution weight crosses .5, the GD schedule starts rotating clockwise with β: the government weighs social consumption more and more, the horizontal intercept of the GD schedule decreases, while the slope increases. The actual GD points in the economy move progressively up–left, from the benchmark point A to points where social consumption increases and the growth rate decreases. At each tick of the redistribution preference β, the growth-consumption equilibrium point moves up to a new GD schedule.

The implication of our results is the following. While it is true that stronger government preferences for redistribution increase social consumption at the expenses of the growth rate, the production technique changes because of the composition of government spending. As a consequence, the terms of the growth–distribution tradeoff also change. A change in government preferences does not produce a movement along a fixed GD schedule. Instead, by progressively changing the technique in use, a stronger preference for redistribution allows the economy to move onto a GD schedule that strictly dominates the benchmark one at higher levels of social consumption: given the chosen value for the growth rate, the level of social consumption per worker is strictly above the one allowed by the benchmark technique. In this respect, the public sector partly alleviates the distributive conflict in the economy.

[FIGURE 2 ABOUT HERE]

4 Labor-Constrained Model

Thus far, we assumed that labor supply was infinitely elastic at the exogenous conventional level of the wage share; in this framework, employment was uniquely determined by labor demand. An alternative model closure would posit the labor force N to grow at the exogenous rate n, and labor supply to be inelastic to the real wage. In a steady state, the labor market equilibrium requires that labor demand grows at the same rate of labor supply. Since labor demand grows with the growth rate of capital stock, we set:

$$g(\tau, \delta) = s(1-\delta)^{\frac{1-\gamma}{\gamma}}(1-\omega)^{\frac{1}{\gamma}}(1-\tau)\tau^{\frac{1-\gamma}{\gamma}} = n. \tag{16}$$

Long-run growth is exogenous and constrained by labor supply. Ever since the pioneering contribution by Harrod (1939), it is customary in the literature to refer to n as the 'natural' growth rate. With the growth rate given at its natural level, the equilibrium condition (16) solves for income distribution, which is now endogenous. Further, since in equilibrium labor demand and labor supply grow at the same rate, the employment rate ($v = L/N$) is constant. While in principle there are two instruments available to policymakers in order to target a value for the labor share, the policy maker does not necessarily need to transfer income to employed workers to integrate the labor share. With growth fixed at the natural rate, taxes are enough to (non linearly) determine income distribution.

Conversely, transfers become relevant when they are used to provide income to the unemployed. The number of unemployed workers in the economy is given by $U = N - L = N(1-v)$. Each unemployed worker receives a subsidy $z = \tau\delta\Pi/U = \tau\delta(1-\omega)Y/U = \tau\delta(1-\omega)L/U = \tau\delta(1-\omega)v/(1-v)$. The interesting implication is that the employment rate becomes a second endogenous variable. Adapting the seminal Goodwin (1967) model to the present framework, we assume that real wage growth depends positively on the employment rate and of the level of unemployment subsidy; indeed, both variables strengthen workers' bargaining power. Accordingly, we posit $\dot{w}/w = f(v, z)$, with $f_v, f_z > 0$. In steady state,

$$f(v, z) = 0. \tag{17}$$

Total differentiation of the system made up of (16) and (17) yields

$$\begin{cases} d\omega = -\frac{(1-\gamma)(1-\omega)}{(1-\delta)} d\delta + (1-\omega)\frac{-\gamma\tau+(1-\gamma)(1-\tau)}{\tau(1-\tau)} d\tau \\ dv\left[\frac{f_v}{f_z} + \frac{\tau\delta(1-\omega)}{(1-v)^2}\right] = -\tau(1-\omega)\frac{v}{1-v}\left[1 + \frac{(1-\gamma)\delta}{(1-\delta)}\right] d\delta - \delta(1-\omega)\frac{v}{1-v}\frac{\gamma}{(1-\tau)} d\tau \end{cases} . \tag{18}$$

Hence, around the steady state, we have the following comparative dynamics results:

$$\begin{cases} \frac{d\omega}{d\delta} < 0; & \frac{d\omega}{d\tau} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr 1-\gamma \\ \frac{dv}{d\delta} < 0; & \frac{dv}{d\tau} < 0 \end{cases} .$$

A few points are worth noting. A higher share of tax revenue that funds unemployment benefits (δ) unambiguously reduces the employment rate and the labor share. From (16), growth is fixed to its natural level; a higher δ produces a reduction in capital accumulation, which has to be compensated by a rise in profits. At the same time, both the higher profit share and the higher δ raise unemployment subsidy. A high z puts upward pressure on wage growth, which need be compensated by a fall in the employment rate.

As of the effect of a change in the tax rate, notice that the growth maximizing condition in the conventional wage share model becomes a wage share maximizing condition. Growth is fixed, and distribution adjusts to make the growth rate of capital equal to the natural growth rate; the tax rate which maximizes capital growth requires the biggest reduction in the profit share, thus maximizing the wage share. The effect on the employment rate is negative. The subsidy z depends on τ both directly as a positive linear function, and, indirectly through ω, as a non monotonic function. The former effect is dominant: an increase in τ raises z, so that a reduction in employment is needed to keep the wage share constant.

The wage share, however, is not the interesting endogenous variable to evaluate policy effects on workers' economic conditions. Total income accruing to workers consists of the sum of wages and unemployment benefits; once the wage share is adjusted to take unemployment benefits into account it has the same functional form as the after tax wage share in the conventional growth model $\tilde{\omega} = \omega + \delta\tau(1-\omega)$. The after tax wage share depends non-monotonically on both policy instruments. In fact, it is easy to show that

$$\begin{cases} \frac{d\tilde{\omega}}{d\delta} \gtreqless 0 \Leftrightarrow \tau \gtreqless \frac{1-\gamma}{1-\gamma\delta} \\ \frac{d\tilde{\omega}}{d\tau} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr (1-\gamma) + \delta\tau(1-\tau) \end{cases}.$$

A higher δ improves workers' overall conditions only if the tax rate is sufficiently large; otherwise, the increase in unemployment benefits cannot offset the negative effect on the labor share due to the lower capital accumulation. On the other hand, since total unemployment benefits are a positive function of taxes, the 'adjusted' wage share is maximized at higher tax rate than the wage share maximizing one.

Notice the overall difference between the conventional wage share and the labor-constrained models. In both models we have two policy instruments, which determine the total tax revenue and its allocation between provision of public good and transfers to workers. In the former model, these policy variables can be used to target output growth and distribution; in the latter they affect distribution and the employment rate.

4.1 Government Preferences and Policy

Differently from the conventional wage share model, the growth rate of the economy is fixed at the natural rate and cannot be manipulated by policy action. For this reason we drop the growth rate from the arguments of the policy function and we substitute it with the unemployed workers' share of output. Letting $\vartheta \in [0, 1]$ denote the weight of the welfare of unemployed workers in the government's preferences, we have the following problem to be solved:

$$\begin{array}{lll} \text{Choose } (\delta, \tau) & \text{to max} & (1-\vartheta)\ln \omega(\delta,\tau) + \vartheta \ln \delta\tau[1-\omega(\delta,\tau)] \\ & \text{s.t.} & (\delta,\tau) \in [0,1]\times[0,1] \end{array} \tag{19}$$

where from (16) the equilibrium level of the wage share is $\omega(\tau, \delta) = \frac{(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}-(n/s)^{\gamma}}{(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}}$. In the Appendix, we derive the first order necessary conditions which, starting with the transfer, read:

$$\delta = \frac{\vartheta\left[(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}-(n/s)^{\gamma}\right]}{\vartheta\left[(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}-(n/s)^{\gamma}\right]+(1-\gamma-\vartheta)(n/s)^{\gamma}}, \tag{20}$$

whereas the first order necessary condition with respect to the tax rate is:

$$\tau = (1-\gamma-\vartheta) + \vartheta\gamma(s/n)^{\gamma}\left[(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}(n/s)^{\gamma}\right]. \tag{21}$$

We cannot find closed form solutions for δ and τ as functions of ϑ, except for the special case where $\vartheta = 0$. Not surprisingly, when government only cares about employed workers, the optimal level of transfer goes to zero, while the optimal tax rate is the wage share maximizing tax rate $\tau = 1-\gamma$. We proceed with an illustrative numerical analysis to show the equilibrium behavior of the relevant variables as a function of government's preferences. Table 2 displays the chosen values of the tax rate, transfer, and the corresponding labor share and total unemployment income in the economy for increasing values of ϑ. The numerical analysis shows that, due to the nonlinear effect of taxes on the labor share, subsumed in the government's problem, the tax rate and the transfer are increasing but not monotone in the weight on unemployed workers.

[TABLE 2 ABOUT HERE]

5 Conclusion

In this paper, we analyzed the role of the size and composition of government spending in influencing both economic growth and income distribution in an economy with two classes, one of which only owns labor services while the other one owns and accumulates capital assets. The emphasis is on the role of different model *closures*: we presented both an endogenous growth model, with perfectly elastic labor supply at a given wage share, and an exogenous growth model with a fixed labor supply and an adjusting real wage.

In the endogenous model, the comparison with a benchmark case in which no income is transferred to workers, shows that higher transfers occur at the expenses of the growth rate. Yet, government action is not merely confined to picking a point along a given growth–distribution tradeoff. Instead, the public sector can partially alleviate the distributive conflict by moving the economy along techniques that, at higher and higher levels of social consumption, allow for a higher growth rate than it would be attained if the benchmark technique were still in use.

Devising a role of the government in the exogenous model is more complicated. The long-run growth rate is given at its natural level, and policymakers do not have a policy tool that can affect growth. Thus, they can focus only on income distribution. We showed that the wage share is a non monotonic function of the tax rate, and it reaches a maximum at the same tax rate that maximizes growth in the endogenous model. On the other hand, policymakers will deviate from the labor share maximizing tax rate when they include the welfare of unemployed workers among their policy goals. We thus have analyzed the equilibrium outcomes for alternative configurations of fiscal policy when the policy makers preferences are defined both over the income of employed and unemployed workers.

The analysis is partial in that: (i) it focuses on steady states only, and (ii) does not consider the role of government debt. Nonetheless, we take it as a useful first pass at analyzing the role of the public sector on economic growth and income distribution in the long run. More work needs to be done to address the dynamic effects of government spending, and incorporate the role of public deficits and debt, in order to increase the relevance of the framework.

Appendix

Efficient choice of the tax rate at high levels of social consumption

We need to show that $\bar{c} < 1$ for $\tau > 1 - \gamma$. Since $\bar{c} = (1-s)(1-\omega) + \omega + s(1-\omega)\frac{(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} - \tau(1-\tau)^{\frac{\gamma}{1-\gamma}}}{\gamma^{\frac{\gamma}{1-\gamma}} - (1-\tau)^{\frac{\gamma}{1-\gamma}}}$, $\bar{c} < 1$ whenever $\frac{(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} - \tau(1-\tau)^{\frac{\gamma}{1-\gamma}}}{\gamma^{\frac{\gamma}{1-\gamma}} - (1-\tau)^{\frac{\gamma}{1-\gamma}}} < 1$. Hence, we have the following series of inequalities: $(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} - \tau(1-\tau)^{\frac{\gamma}{1-\gamma}} < \gamma^{\frac{\gamma}{1-\gamma}} - (1-\tau)^{\frac{\gamma}{1-\gamma}} \Leftrightarrow (1-\tau)^{\frac{1}{1-\gamma}} < \gamma^{\frac{1}{1-\gamma}} \Leftrightarrow \tau > 1-\gamma$.

First order conditions in the labor-constrained model

Let us start by calculating the derivative of the objective function with respect to δ, and by setting it equal to zero. We find

$$\frac{(1-\vartheta)}{\omega}\frac{d\omega}{d\delta} = -\vartheta\frac{\left[(1-\omega) - \delta\frac{d\omega}{d\delta}\right]}{\delta(1-\omega)}.$$

Substitute $d\omega/d\delta = -\frac{(1-\gamma)(1-\omega)}{(1-\delta)}$ from (18) and $\omega = \frac{(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma} - (n/s)^{\gamma}}{(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}}$ into the previous condition, and rearrange to find (20).

Next, find the derivative of the objective function with respect to τ, and set it equal to zero to obtain

$$\frac{(1-\vartheta)}{\omega}\frac{d\omega}{d\tau} = -\vartheta\frac{\left[(1-\omega) - \tau\frac{d\omega}{d\tau}\right]}{\tau(1-\omega)}.$$

Substitute $d\omega/d\tau = (1-\omega)\frac{-\gamma\tau + (1-\gamma)(1-\tau)}{\tau(1-\tau)}$ from (18) and $\omega = \frac{(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma} - (n/s)^{\gamma}}{(1-\delta)^{1-\gamma}(1-\tau)^{\gamma}\tau^{1-\gamma}}$ into the previous condition, and rearrange to find (21).

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A Tables and Figures

β	$\in [0, .5]$	.6	.7	.8	.9
τ	.25	.272723	.426706	.588359	.78272
δ	0	.111091	.552155	.766786	.87855
g_K	.1775	.169964	.121046	.074258	.0293533
c	.52	.535355	.639678	.75	.871436

Table 1: Choice of δ, τ for increasing values of the preference parameter β.

θ	0	.2	.4	.6	.8
τ	.25	.703225	.700955	.90507	.90507
δ	0	.859327	.995274	.930797	.930797
ω	.884203	.707363	.319821	.2287	.2287
$\tau\delta(1-\omega)$	0	.176841	.474522	.649772	.649772

Table 2: Choice of δ, τ for increasing values of θ, the weight on unemployment income in government's preferences.

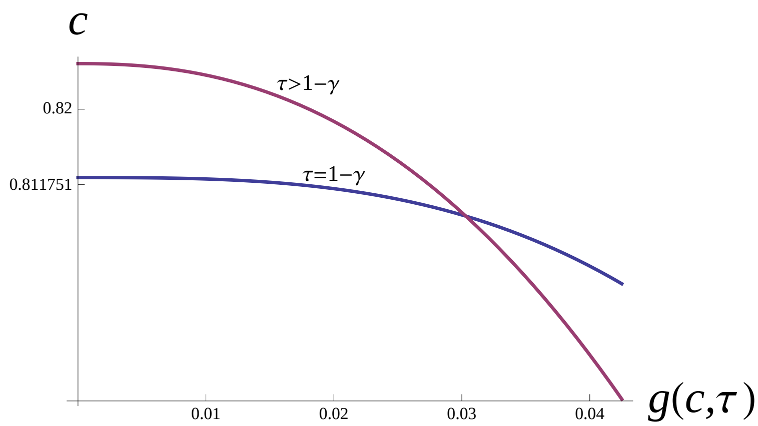

Figure 1: Conventional wage share model: growth-distribution frontiers. At high levels of social consumption, the growth maximizing tax rate does not attain the most efficient growth rate.

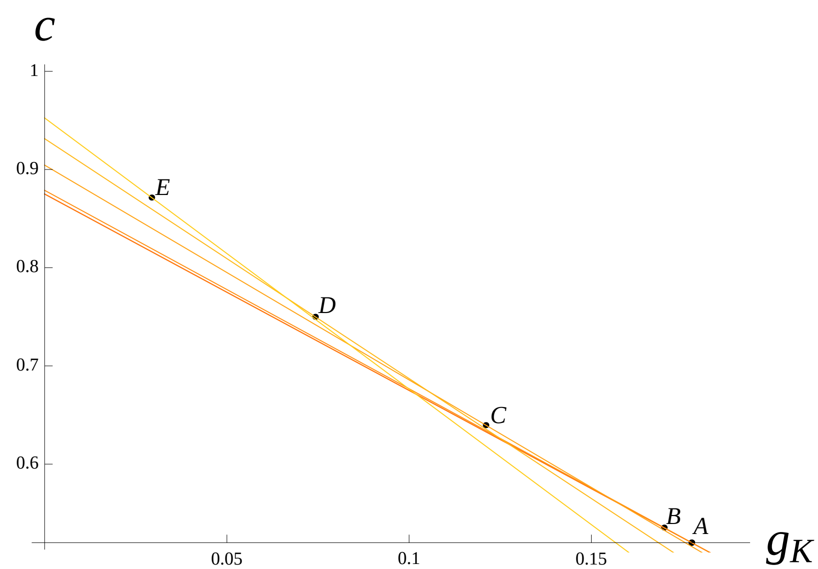

Figure 2: Conventional wage share model: different GD schedules and GD points for changing values of the distribution weight β. The dark orange line is the benchmark case with zero transfer. Lighter tones correspond to increasing distribution weights.